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BOUNDARY INTEGRAL EQUATIONS FOR PLANE ORTHOTROPIC BODIES – A MODIFICATION FOR EXTERIOR REGIONS

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Abstract: *Assuming linear displacements and constant strains and stresses at infinity, we reformulate the equations of the direct boundary element method for plane problems of elasticity. We consider a body made of orthotropic material. The reformulated equations make it possible to attack plane problems on exterior regions without replacing the region by a bounded one.*

Keywords: *Boundary element method, Somigliana relations, direct method, orthotropic body, plane problems, exterior regions*

1. INTRODUCTION

This sample paper does not contain details concerning the format requirements. Instead the reader is recommended to rewrite and retype it with MS Word or in a LaTeX environment. The text is taken from a paper with a special attention on the format requirements presented in the Hungarian sample papers.

A large literature studies plane problems for orthotropic bodies, including [1, 2], as well as the books [3, 4] and the references therein. However, as explained by [5], the standard formulation for exterior regions has the disadvantage that no stresses can be prescribed at infinity.

2. GOVERNING EQUATIONS

For homogenous and orthotropic material the plane problem of classical elasticity is governed by the kinematic equations

$$e_{\rho\lambda} = \frac{1}{2}(\partial_\rho u_\lambda + u_\rho \partial_\lambda), \tag{1}$$

Hooke's law

$$\begin{aligned} t_{11} &= c_{11}e_{11} + c_{12}e_{22}, \\ t_{22} &= c_{12}e_{11} + c_{22}e_{22}, \\ t_{12} &= t_{21} = 2c_{66}e_{12}, \end{aligned} \tag{2}$$

where

$$c_{11} = \frac{s_{22}}{d}, \quad c_{12} = c_{21} = -\frac{s_{12}}{d}, \quad c_{22} = \frac{s_{11}}{d}, \quad c_{66} = \frac{1}{s_{66}}, \quad d = s_{11}s_{22} - s_{12}^2, \tag{3}$$

and the equilibrium equations

$$t_{\rho\lambda}\partial_\lambda + b_\rho = 0 \tag{4}$$

which should be associated with appropriate boundary conditions. The basic equation for u_λ takes the form

$$\mathcal{D}_{\rho\lambda} u_\lambda + \frac{b_\rho}{\mu} = 0, \tag{5}$$

where

$$[\mathcal{D}_{\rho\lambda}] = \begin{bmatrix} c_{11}\partial_1^2 + c_{66}\partial_2^2 & (c_{12}+c_{66})\,\partial_1\partial_2 \\ (c_{21}+c_{66})\,\partial_2\partial_1 & c_{22}\partial_2^2 + c_{66}\partial_1^2 \end{bmatrix}. \tag{6}$$

Let $Q(\xi_1, \xi_2)$ and $M\,(x_1, x_2)$ be two points in the plane (the source point and the point of effect). We shall assume temporarily that the point *Q* is fixed. The distance between *Q* and *M* is *R*, the position vector of *M* relative to *Q* is r_κ. The small circle as a subscript (for instance $M_\circ$ or $Q_\circ$) indicates that the corresponding points, i.e., *Q* or *M* are taken on the contour.

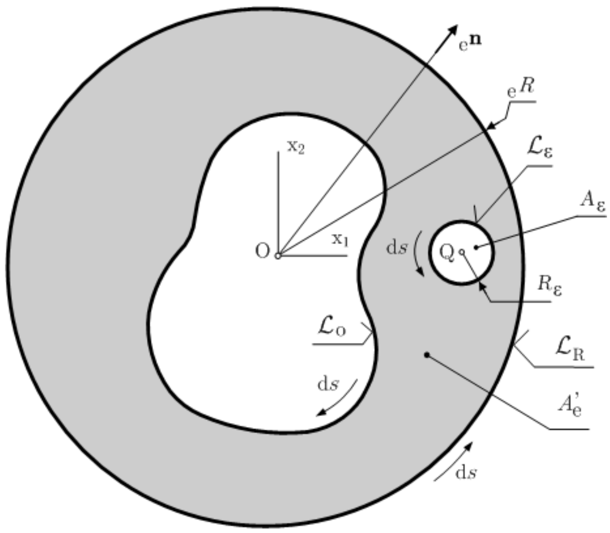

Figure 1. An exterior region

3. EXAMPLES

Lekhtniski's book [6] contains closed form solutions for the stresses on the boundary, as well as numerical values which can be found in Table 17 on page 197. In this paper we show the results as computed by solving the corresponding boundary integral equation and the results taken from [6]. We used a polar coordinate system, and the table contains the quotients σ_r / p, $\tau_{r\theta} / p$ and σ_θ / p for the plane with the circular inclusion.

Rigid inclusion						
Polar angle	σ_r/p	[6], p.197	$\tau_{r\theta}/p$	[6], p.197	σ_θ/p	[6], p.197
0^o	1.2363	1.237	0.0000	0.0000	0.0445	0.044
30^o	0.9370	0.937	-0.5185	-0.519	0.2697	0.270
60^o	0.3383	0.388	-0.5185	-0.519	0.6989	0.699
90^o	0.0389	0.039	0.0000	0.000	0.0028	0.003

Table 1. Results for the rigid inclusion

We used partially discontinuous elements of order two. The results obtained are in good agreement with those taken from Lekhtniski's book [6].

4. CONCLUDING REMARKS

We have modified the Somigliana formulas for exterior regions of orthotropic bodies by assuming that the strains are constants and accordingly the displacements are linear at infinity. Under this condition the strain energy density is bounded (although the strain energy is not), and there is no need to replace the exterior region by a finite one if a constant stress condition is prescribed at infinity. This can be an advantage if one considers an infinite plane with holes or cracks subjected to constant stresses at infinity, and an attempt is made to determine the stresses in finite. Existing codes can be modified easily to perform computations. We also presented illustrative results for two simple problems.

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