

NUMERICAL SOLUTIONS FOR STEADY VISCOUS FLOW PAST A CIRCULAR CYLINDER IN AN ALIGNED MAGNETIC FIELD

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[Received: March 26, 2004, Accepted: July 16, 2004.]

Abstract. The steady incompressible magneto-hydrodynamic (MHD) flow past a circular cylinder with an aligned magnetic field is studied for the Reynolds number (Re) up to 40, using the Hartmann number, M as the perturbation parameter. The multigrid method with defect correction technique is used to achieve the second order accurate solution of complete nonlinear Navier-Stokes equations. The magnetic Reynolds number is assumed to be small. It is observed that as M is increased the volume of the separation bubble decreases and drag coefficient increases. The graphs of streamlines, vorticity lines, surface pressure, surface vorticity and drag coefficient are presented and the effect of the magnetic field is discussed.

Mathematical Subject Classification: 76DXX, 76005, 76W05

Keywords: Navier-Stokes equations, MHD, Hartmann number, Multigrid method, Defect correction.

1. INTRODUCTION

In the plane flow of an incompressible, viscous, electrically conducting fluid over a solid body, the presence of a normal magnetic field at the surface has the effect of alleviating an adverse pressure gradient. One might expect that separation of the boundary layer would be delayed as a result. Indeed, since the degree to which the unfavorable pressure gradient is alleviated depends on the strength of the magnetic field, it is conceivable that separation could be completely suppressed. In a two dimensional flow of a liquid with small magnetic Reynolds number under the influence of an applied magnetic field, it has been proved that if the field is strong enough and

has the appropriate orientation, then separation of a viscous boundary layer can be prevented even up to rear stagnation point.

The problem of steady two-dimensional incompressible MHD flow past a circular cylinder with an applied magnetic field parallel to the main flow was investigated by Bramely [5] using the Oseen approximation. He later extended the problem to full Navier-Stokes equations for low Reynolds numbers using the method of series truncation [6]. Shanti Swarup and Sinha [22] investigated the steady flow of an incompressible, viscous, electrically conducting fluid past a non-magnetic and non-conducting circular cylinder for low Reynolds numbers [$Re = O(R_m)$ and $R_m \ll M \ll 1$], using the method of matched asymptotic expansions. Very recently, Sekhar *et al.* [21] studied the MHD flow past a sphere in an aligned magnetic field using the multigrid method.

In the absence of a magnetic field the present problem corresponds to the steady, viscous flow past a circular cylinder which has been studied by several researchers, including Dennis and Chang [8], Fornberg [9, 10], Kawamura and Kuwahara [15], Braza *et al.* [7], Karniadakis and Triantafyllou [14], Ingham and Tang [11] and Baranyi and Shirakashi [2]. Roshko [20], Bearman [4] and Norberg [16, 17] have studied the flow past a cylinder experimentally and have provided invaluable data. It is well known that the flow around a cylinder becomes unstable at $Re \sim 49$ due to periodic vortex shedding [25]. Bae *et al.* [1] investigated the conditions needed to facilitate the suppression of Kármán vortex excitation of a circular cylinder by a second cylinder set downstream in a cruciform arrangement. Baranyi [3] studied the unsteady momentum and heat transfer from a fixed cylinder using the finite difference method. In this paper, we discuss the flow of a conducting fluid past a circular cylinder for a range of Reynolds numbers from 10 to 40 and for the intermediate values of Hartmann number M using the finite difference method. The multigrid method with defect correction technique is applied to obtain the second order accurate solution.

2. MATHEMATICAL MODELLING

The equations governing the steady MHD flow of an incompressible fluid (with finite electrical conductivity σ) past a circular cylinder (of radius a) with uniform free-stream velocity $\mathbf{U}_\infty$ and an uniformly applied magnetic field $\mathbf{H}_\infty$ at large distances are, in non-dimensional form:

Momentum equation

$$\frac{Re}{2}(\mathbf{q} \cdot \nabla)\mathbf{q} = -\nabla p + \nabla^2 \mathbf{q} + \frac{M^2}{2R_m}[(\nabla \times \mathbf{H}) \times \mathbf{H}] \quad (1)$$

Ohm's law

$$\mathbf{j} = (\nabla \times \mathbf{H}) = \frac{R_m}{2}[\mathbf{E} + \mathbf{q} \times \mathbf{H}] \quad (2)$$

Equation of continuity

$$\nabla \cdot \mathbf{q} = 0 \quad (3)$$

where p is the pressure, $\mathbf{q}$ the fluid velocity, $\mathbf{H}$ the magnetic field, $\mathbf{E}$ the electric field, $\mathbf{j}$ the current density. The Reynolds number is $Re = 2\rho U_\infty a/\eta$ and $M = \mu H_\infty a(\sigma/\eta)^{1/2}$ is the Hartmann number. The magnetic Reynolds number is given by $R_m = U_\infty a \mu \sigma$. The viscosity, density and magnetic permeability of the fluid are η , ρ and μ respectively. The following non-dimensional terms were substituted to obtain the dimensionless differential equations:

$$\mathbf{q} = \frac{\mathbf{q}'}{U_\infty}, \quad p = \frac{a}{\rho \nu U_\infty} p', \quad r = \frac{r'}{a}, \quad \mathbf{H} = \frac{\mathbf{H}'}{H_\infty}$$

$$\mathbf{E} = \frac{\mathbf{E}'}{E_\infty}, \quad \mathbf{j} = \frac{\mathbf{j}'}{j_\infty}$$

where primed variables are dimensional quantities and ν is the kinematic viscosity, E_∞ and j_∞ are the magnitudes of electric field intensity and current density at infinity, respectively. In order to satisfy equation (2.3), the dimensionless stream function $\psi(r, \theta)$ is introduced such that

$$u = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad v = -\frac{\partial \psi}{\partial r} \quad (4)$$

where u and v are the dimensionless radial and transverse components of fluid velocity. Cylindrical polar coordinates (r, θ, z) are used in such a way that the flow is symmetric about $\theta = 0^\circ$ and $\theta = 180^\circ$. As the magnetic field and fluid flow are aligned at infinity, the electric field can be assumed to be zero. The problem can be simplified by assuming the magnetic Reynolds number to be small. The magnetic field must not be so large that the flow develops into a slug flow. It should be large enough to see the effect on separation but small enough for the flow to be a perturbation of the potential flow with zero magnetic field. We use the low- R_m approximation and ignore equation (2.2) as well as replace the magnetic field in all MHD equations by

$$\mathbf{H} = (-\cos \theta, \sin \theta, 0), \quad (5)$$

which will eliminate several nonlinear terms of unknown quantities in the governing equations. After eliminating pressure from equation (2.1), we get

$$\frac{Re}{2} [\nabla \times (\boldsymbol{\omega} \times \mathbf{q})] = \nabla^2 \boldsymbol{\omega} + \frac{M^2}{2R_m} [\nabla \times \{(\nabla \times \mathbf{H}) \times \mathbf{H}\}] \quad (6)$$

where

$$\boldsymbol{\omega} = \nabla \times \mathbf{q} \quad (7)$$

is the vorticity. Substitution of equation (2.2) in equation (2.6) gives

$$\nabla^2 \boldsymbol{\omega} = \frac{Re}{2} [\nabla \times (\boldsymbol{\omega} \times \mathbf{q})] - \frac{M^2}{4} [\nabla \times \{(\mathbf{q} \times \mathbf{H}) \times \mathbf{H}\}] \quad (8)$$

Expanding equations (2.7) and (2.8) we get

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = -\omega \quad (9)$$

and

$$\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \omega}{\partial \theta^2} - \frac{Re}{2r} \left[\frac{\partial \psi}{\partial \theta} \frac{\partial \omega}{\partial r} - \frac{\partial \psi}{\partial r} \frac{\partial \omega}{\partial \theta} \right] = \frac{M^2}{4} \left[\omega \sin^2 \theta + \frac{\sin 2\theta}{r} \frac{\partial^2 \psi}{\partial r \partial \theta} - \frac{\sin 2\theta}{r^2} \frac{\partial \psi}{\partial \theta} - \cos 2\theta \frac{\partial^2 \psi}{\partial r^2} \right] \quad (10)$$

Since major velocity gradients occur near the body, we used the transformations $r = e^{\pi \xi}$ and $\theta = \pi \eta$ to concentrate the mesh spacing near the body. Then equations (2.9) and (2.10) can be written as

$$\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2} + \pi^2 e^{2\pi \xi} \omega = 0 \quad (11)$$

and

$$\begin{aligned} \frac{\partial^2 \omega}{\partial \xi^2} + \frac{\partial^2 \omega}{\partial \eta^2} - \frac{Re}{2} \left[\frac{\partial \psi}{\partial \eta} \frac{\partial \omega}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial \omega}{\partial \eta} \right] = \frac{M^2}{4} \left[\pi^2 e^{2\pi \xi} \omega \sin^2(\pi \eta) + \sin 2(\pi \eta) \frac{\partial^2 \psi}{\partial \xi \partial \eta} \right. \\ \left. - \pi \sin 2(\pi \eta) \frac{\partial \psi}{\partial \eta} - \cos 2(\pi \eta) \frac{\partial^2 \psi}{\partial \xi^2} + \pi \cos 2(\pi \eta) \frac{\partial \psi}{\partial \xi} \right] \end{aligned} \quad (12)$$

in the vorticity-stream function form. Equations (2.11) and (2.12) must now be solved subject to the following boundary conditions:

(1) On the surface of the cylinder; ($r = 1$), $\xi = 0$,

$$\psi = \frac{\partial \psi}{\partial \xi} = 0$$

$$\omega = -\frac{1}{\pi^2} \frac{\partial^2 \psi}{\partial \xi^2}$$

(2) At large distances from the cylinder; ($r \rightarrow \infty$), $\xi \rightarrow \infty$,

$$\psi \sim e^{\pi \xi} \sin(\pi \eta)$$

$$\omega \rightarrow 0$$

(3) Along the axis of symmetry ($\eta = 0, \eta = 1$), $\psi = 0$ and $\omega = 0$.

3. NUMERICAL METHOD

The coupled nonlinear Navier-Stokes equations are solved by first applying finite difference method and the resulting algebraic equations are solved by using the multigrid method. Here, a recursive multigrid procedure is employed in which the smoother is a point Gauss Seidel iteration and the usual coarse grid correction is applied as follows [23]. Let there be a sequence of computational grids $G^1, G^2, \dots, G^l$ with G^k finer than G^{k-1} . Let $U^k \rightarrow \mathbf{R}$ be the space of grid functions on G^k , let $P^k : U^{k-1} \rightarrow U^k$ be a prolongation operator and let $R^k : U^{k+1} \rightarrow U^k$ be a restriction operator. Suppose

we have a nonlinear (system of) partial differential equation(s), discretized on $G^1, G^2, \dots, G^l$. On G^k , the algebraic problem to be solved is given by

$$A^k(u^k) = f^k$$

where, A^k is the matrix obtained by suitable discretization. If $\hat{u}$ is the approximation to exact solution u , then, $(\hat{u} - u)$ represents the error e . Then, we have

$$Ae = -r = A\hat{u} - f \quad (13)$$

where, r is called the residue. The coarse grid approximation $\bar{u}$ of $-e$ satisfies

$$\bar{A}\bar{u} = Rr$$

where, $\bar{A}$ is the operator obtained by discretizing the original problem on a coarser grid and R is the restriction operator. If the grid under consideration is coarsest, then the above equation should be solved exactly. The coarse grid correction to be added to $\hat{u}$ is $P\bar{u}$ (where P is the prolongation operator) given by

$$\hat{u} = \hat{u} + P\bar{u}$$

This represents one multigrid cycle. Solving on G^{l-1} by γ multigrid iterations results in the following recursive algorithm:

```

procedure MG( $k, u, f$ )
  begin if  $k = 1$  then solve  $A^1(u^1) = f^1$  else
    begin  $S_1(k, u, f)$ 
      Choose  $\tilde{u}^{k-1} \in U^{k-1}$ 
       $\tilde{f}^{k-1} = A^{k-1}(\tilde{u}^{k-1})$ 
       $f^{k-1} = \tilde{f}^{k-1} + R^{k-1}(f^k - A^k(u^k))$ 
      for  $i := 1$  step 1 until  $\gamma$  do MG( $k - 1, u, f$ )
       $u^k = u^k + P^k(u^{k-1} - \tilde{u}^{k-1})$ 
       $S_2(k, u, f)$ 
    end
  end MG

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where, S denotes a smoother involving a small number of point Gauss Seidel iterations.

The initial solution is taken as $\psi = 0$ and $\omega = 0$ at all inner grid points except for ψ at $\xi = \infty$ where the boundary condition holds. In finding the solution for higher values of Re and M , the solution obtained for lower values of Re and M are used as starting solution. Among the two variables, ω and ψ , we first solved for ω and then for ψ . Convergence is said to have been achieved when the difference between two successive iterations m and $m + 1$, at all interior grid points, is less than 10^{-5} , i.e.,

$$|\psi^{m+1} - \psi^m| < 10^{-5}$$

and

$$|\omega^{m+1} - \omega^m| < 10^{-5}.$$

The restriction operator R_k^{k-1} transfers a fine grid function U^k to a coarse grid function U^{k-1} . On the other hand the prolongation operator, denoted as P_{k-1}^k , transfers a coarse grid function U^{k-1} to a fine grid function U^k . For the restriction operator, the simplest form is ‘injection’ where by the values of a function in the coarse grid are taken to be exactly the values at the corresponding points of the next fine grid *i.e.*,

$$(R_k^{k-1}u^k)_{i+1,j+1} = u_{2i+1,2j+1}^k.$$

We used the above injection operator throughout this study. For the prolongation operator the simplest form is derived using linear interpolation. Prolongation by linear interpolation introduces no ambiguity when the interpolated value is desired at the mid points of the boundaries of a mesh cell. The following 9-point prolongation operator defined by Wesseling [24] is used for the present study

$$\begin{aligned} (P_{k-1}^k u^{k-1})_{2i+1,2j+1} &= u_{i+1,j+1}^{k-1} \\ (P_{k-1}^k u^{k-1})_{2i+2,2j+1} &= \frac{1}{2} (u_{i+1,j+1}^{k-1} + u_{i+2,j+1}^{k-1}) \\ (P_{k-1}^k u^{k-1})_{2i+1,2j+2} &= \frac{1}{2} (u_{i+1,j+1}^{k-1} + u_{i+1,j+2}^{k-1}) \\ (P_{k-1}^k u^{k-1})_{2i+2,2j+2} &= \frac{1}{4} (u_{i+1,j+1}^{k-1} + u_{i+2,j+1}^{k-1} + u_{i+1,j+2}^{k-1} + u_{i+2,j+2}^{k-1}). \end{aligned}$$

The solution obtained by the above method is not second order accurate as we have approximated all terms by second order central difference method except convective terms which are approximated by first order upwind difference scheme to ensure diagonal dominance. In order to achieve second order accurate solution, the defect correction method is employed as follows. If $\mathbf{B}$ is the operator obtained, for example, by first order upwind discretization and $\mathbf{A}$ is that obtained by second order accurate discretization, then defect correction algorithm [12, 13] works as given below. At the start of defect correction, $\bar{y}$ is a solution that is not second order accurate, and at the end of defect correction, $\bar{y}$ is second order accurate.

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begin Solve  $\mathbf{B}\bar{y} = b$ 
  for  $i := 1$  step 1 until  $n$  do
    solve  $\mathbf{B}y = b - \mathbf{A}\bar{y} + \mathbf{B}\bar{y}$ 
     $\bar{y} := y$ 
  od
end

```

Usually, in practice, it is sufficient to take $n = 1$ or 2.

4. RESULTS AND DISCUSSIONS

Full, nonlinear Navier-Stokes equations for the MHD flow past a circular cylinder are solved using the multigrid method with defect correction technique for the range of Reynolds numbers from 10 to 40 and for different values of Hartmann number M , using 512×512 as the finest grid with 256×256 , 128×128 and 64×64 as coarser

grids. The finite difference method is applied to the grid shown in Figure 1. We observed the separation at rear stagnation point for all Reynolds numbers considered in the present study, in $M = 0$ case (Figures 2–7). The length of the wake (l/a) and angle of the separation (θ_s) are found to increase with Re as observed by some researchers [8, 9]. With no magnetic field ($M = 0$), both length of the wake and angle of the separation values (Tables 1–3) are in good agreement with Dennis and Chang [8]. We observed that as the magnetic field is increased, the Lorentz forces dominate and produce a convective rate in a direction opposite to the flow resulting in the decrease of wake length and separation angle, for all Re values ($10 \leq Re \leq 40$). A similar phenomenon can be seen in the case of the translation of a sphere in a rotating viscous fluid [18, 19] and MHD flow past a sphere [21]. For $Re = 10$ the separation bubble disappeared completely at $M = 3$ (Figure 3). As the magnetic forces are proportional to and resist the flow of fluid in any other direction than that of the unperturbed magnetic field, near the cylinder, they produce changes in the pattern of the vorticity lines. The length of standing vortex is reduced slightly and the strength of the disturbance in front of the cylinder is increased with increasing magnetic field (Figures 8–13). It can also be seen that the radial component (u) of fluid velocity near the cylinder at $\theta = 90^\circ$ is suppressed more compared to the transverse component (v) as it (u) is not parallel to the magnetic field (Figures 16, 17). As the Hartmann number increases, the thickness of the boundary layer adjoining the cylinder surface decreases, indicating that it tends to zero for sufficiently large values of M ($M \gg 1$) (Figure 16). This may be attributed to the enhanced velocity gradients required by the viscous stresses to compete with the large magnetic forces.

The drag coefficients and surface pressure are calculated using the following relations:

Viscous drag coefficient

$$C_v = -\frac{4\pi}{Re} \int_0^1 \omega_{\xi=0} \sin(\pi\eta) d\eta \quad (14)$$

Pressure drag coefficient

$$C_p = \frac{4}{Re} \int_0^1 \left(\frac{\partial \omega}{\partial \xi} \right)_{\xi=0} \sin(\pi\eta) d\eta \quad (15)$$

Total drag coefficient

$$C_D = C_v + C_p \quad (16)$$

and surface pressure

$$P(0, \eta) = 1 - \frac{4}{\pi Re} \int_0^\infty \left(\frac{\partial \omega}{\partial \eta} \right)_{\eta=1} d\xi - \frac{4}{Re} \int_\eta^1 \left(\frac{\partial \omega}{\partial \xi} \right)_{\xi=0} d\eta \quad (17)$$

We found that as the thickness of the boundary layer decreases, the increased velocity gradients at the surface will increase the pressure drop (Figure 14) necessary to maintain the given flow rate. It can be seen from Figure 15 that the magnetic field tends to suppress the surface vorticity behind the cylinder thereby competing with the viscous diffusion of vorticity out from the surface. The observed flow field

is in accordance with the assumption that the effect of magnetic field is the small perturbation of zero field potential flow.

Table 1. Drag coefficient values for $Re = 10$

M	256×256			512×512			θ_s	$P(0,0)$	$P(0,1)$	(l/a)
	C_v	C_p	C_D	C_v	C_p	C_D				
0.00	1.23	1.57	2.80	1.24	1.58	2.82	29.40	-0.70	1.48	1.55
1.00	1.23	1.61	2.84	1.23	1.59	2.83	25.40	-0.73	1.48	1.34
2.00	1.23	1.70	2.93	1.23	1.70	2.93	14.06	-0.83	1.50	1.05
3.00	1.30	1.93	3.23	1.31	1.93	3.24	0	-1.06	1.53	0

Table 2. Drag coefficient values for $Re = 20$

M	256×256			512×512			θ_s	$P(0,0)$	$P(0,1)$	(l/a)
	C_v	C_p	C_D	C_v	C_p	C_D				
0.00	0.79	1.20	1.99	0.80	1.22	2.02	43.80	-0.53	1.26	2.81
3.00	0.79	1.26	2.05	0.79	1.26	2.05	34.71	-0.60	1.27	1.80
5.00	0.82	1.54	2.36	0.82	1.54	2.36	26.36	-0.87	1.30	1.40
7.00	0.89	1.90	2.79	0.89	1.90	2.79	21.72	-1.50	1.35	1.25

Table 3. Drag coefficient values for $Re = 40$

M	256×256			512×512			θ_s	$P(0,0)$	$P(0,1)$	(l/a)
	C_v	C_p	C_D	C_v	C_p	C_D				
0.00	0.51	0.97	1.48	0.51	0.97	1.48	53.59	-0.46	1.14	5.74
3.00	0.51	1.00	1.51	0.51	1.01	1.52	44.45	-0.47	1.14	3.81
5.00	0.53	1.15	1.68	0.53	1.15	1.68	36.01	-0.54	1.15	2.80
7.00	0.57	1.43	2.00	0.57	1.44	2.01	31.09	-0.68	1.15	2.30

Table 4. Drag coefficient values before and after defect correction for $M = 0$

Re	128×128		256×256		512×512	
	before DC	after DC	before DC	after DC	before DC	after DC
10	2.7186	2.7280	2.7561	2.7683	2.8133	2.8221
20	1.8826	1.8901	1.9905	1.9941	1.9954	2.0180
40	1.3851	1.3928	1.4644	1.4723	1.4742	1.4805

Table 5. Comparison of Drag coefficient values for $M = 0$

Re	Present results	Dennis and Chang [8]	Fornberg [9]	Ingham and Tang [11]
10	2.82	2.846	—	—
20	2.02	2.045	2.000	1.995
40	1.48	1.522	1.498	—

In the case of $M = 0$, the drag coefficient values are in good agreement with the earlier work [8, 9, 11]. The drag coefficient values before and after applying defect correction (DC) in three different grids 128×128 , 256×256 and 512×512 are given

in Table 4. The comparison of the drag coefficient values for $M = 0$ is given in Table 5 and the graph of drag coefficient versus Reynolds number is presented in Figure 18. It is observed that the effect of magnetic field decreases as Re increases up to the range considered in this study.

APPENDIX A. FIGURES

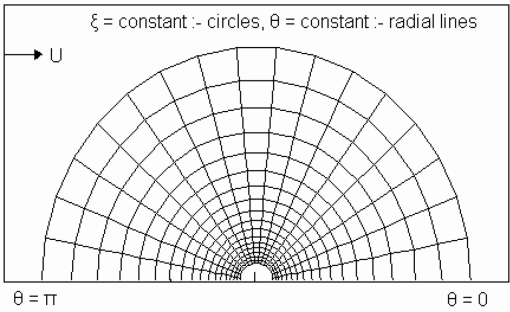


Figure 1. Finite difference grid

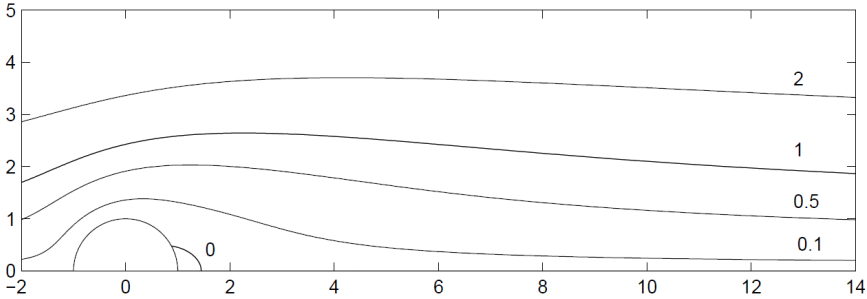


Figure 2. Streamlines for $Re = 10$, $M = 0$

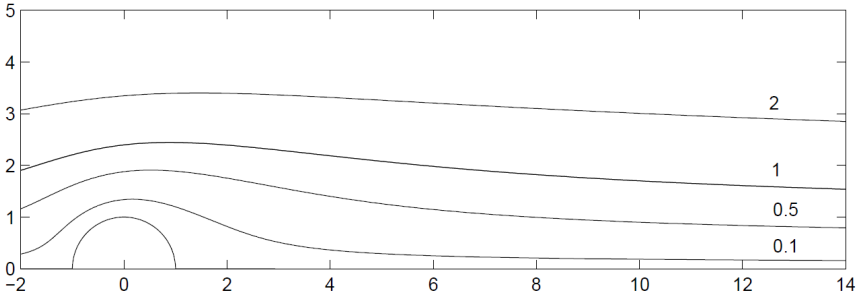
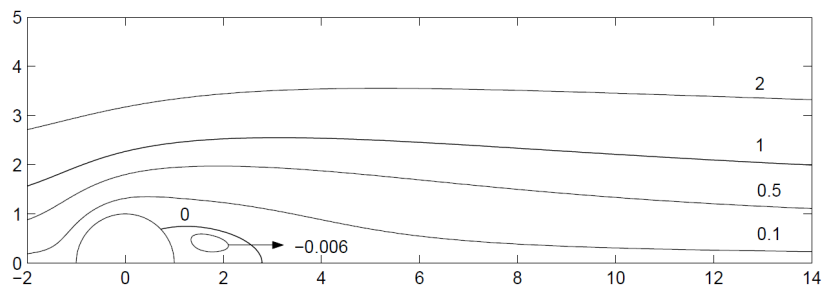
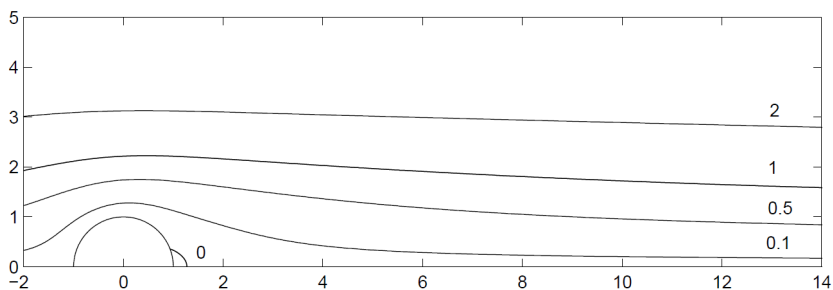
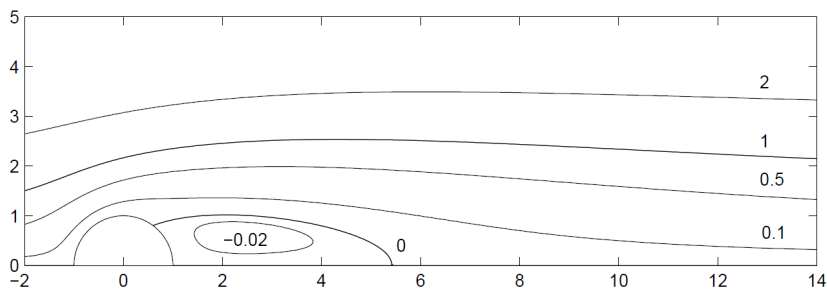
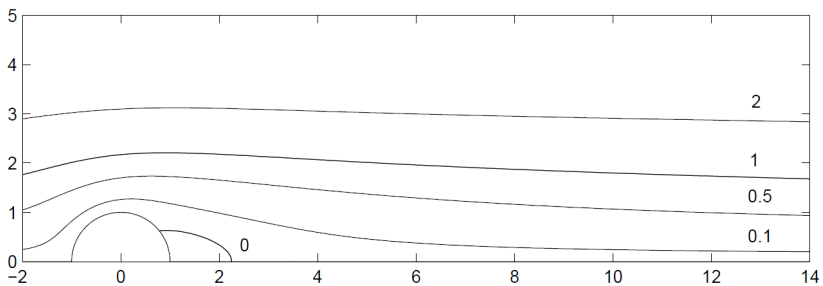


Figure 3. Streamlines for $Re = 10$, $M = 3$

Figure 4. Streamlines for $Re = 20$, $M = 0$ Figure 5. Streamlines for $Re = 20$, $M = 7$ Figure 6. Streamlines for $Re = 40$, $M = 0$ Figure 7. Streamlines for $Re = 40$, $M = 7$

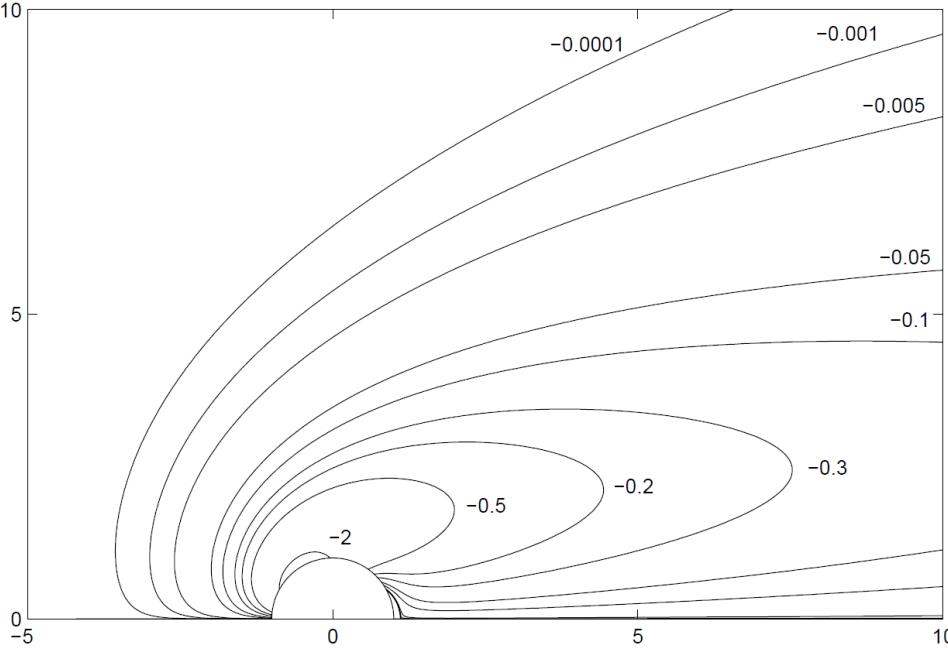


Figure 8. Vorticity lines for $Re = 10$, $M = 0$

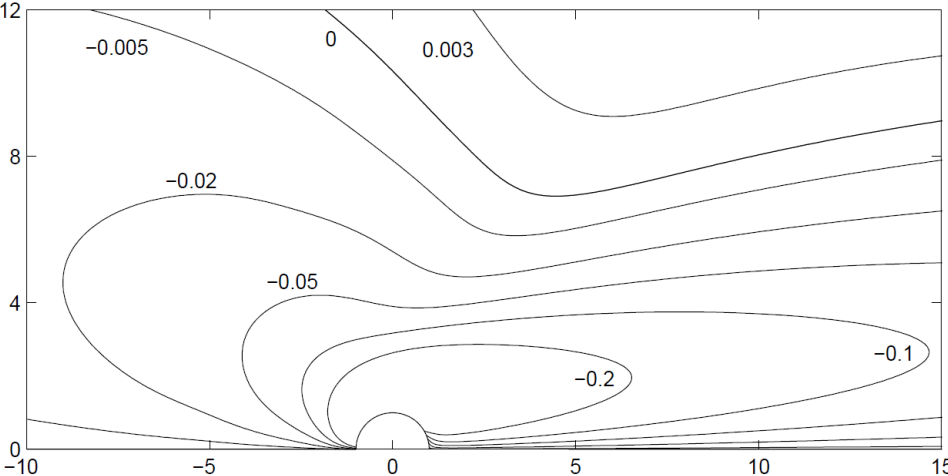
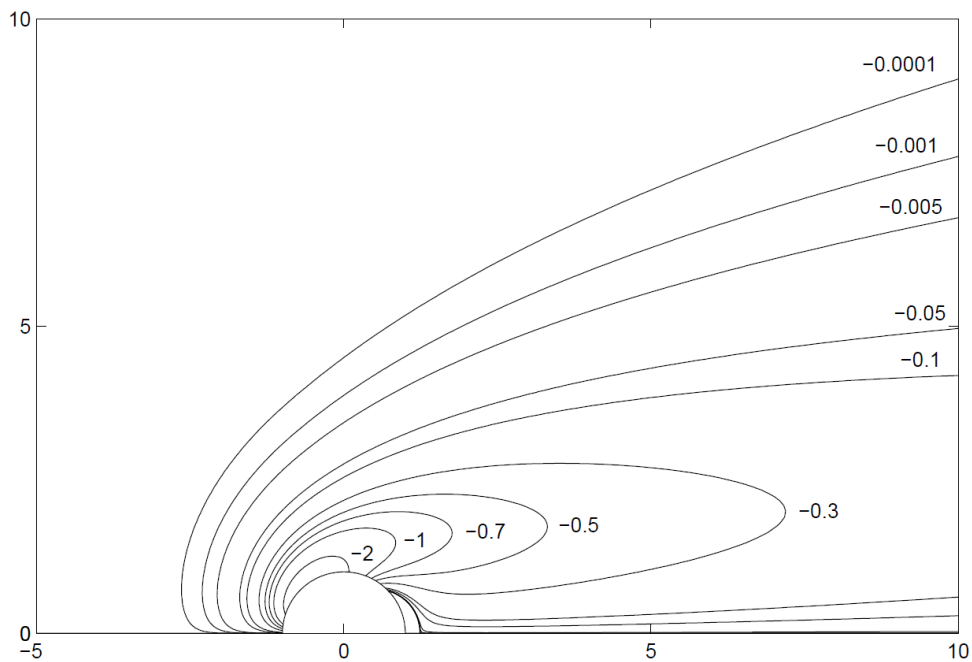
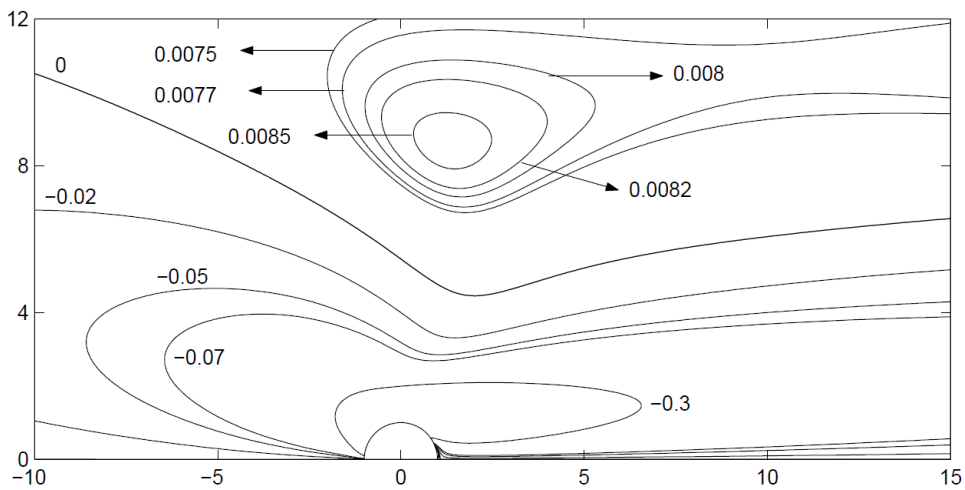


Figure 9. Vorticity lines for $Re = 10$, $M = 3$

Figure 10. Vorticity lines for $Re = 20$, $M = 0$ Figure 11. Vorticity lines for $Re = 20$, $M = 7$

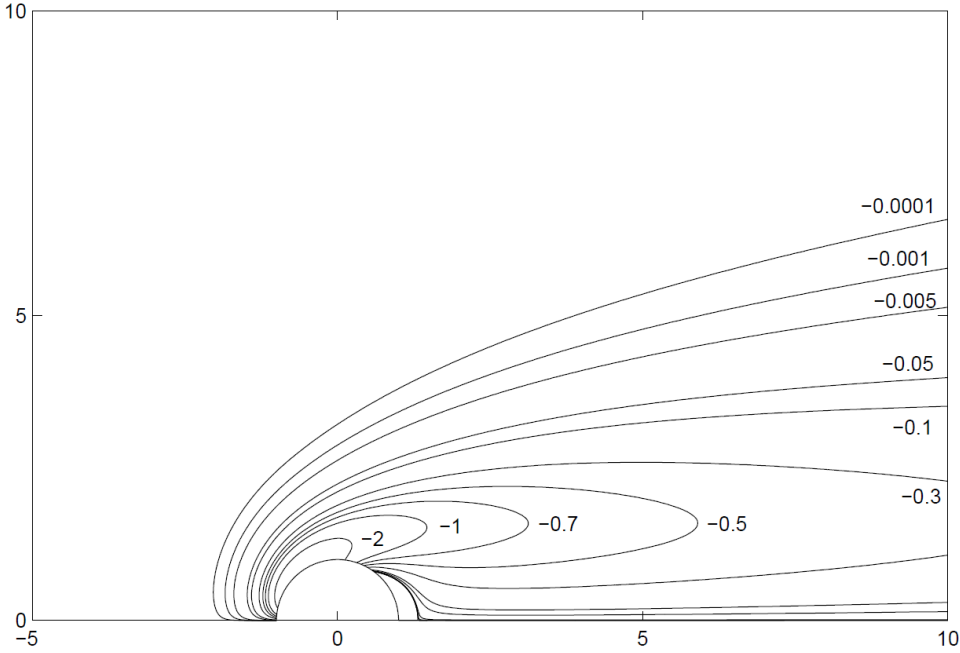


Figure 12. Vorticity lines for $Re = 40$, $M = 0$

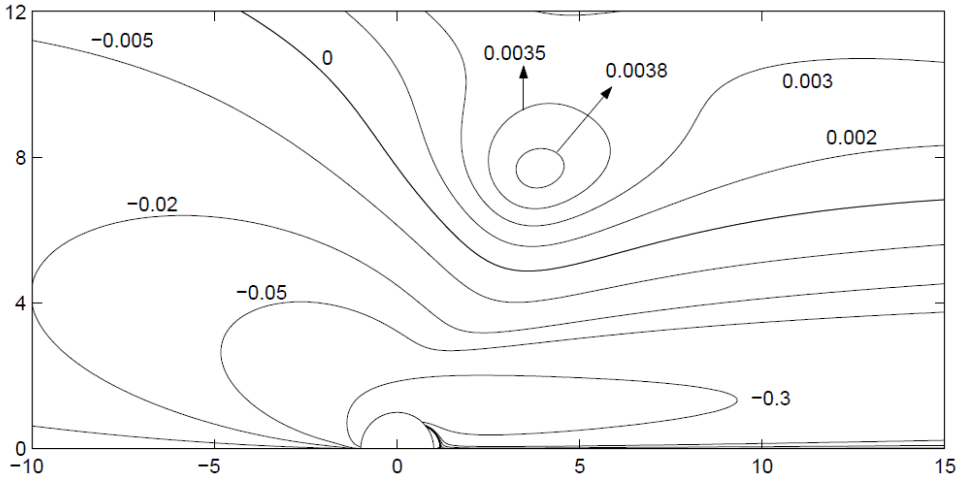


Figure 13. Vorticity lines for $Re = 40$, $M = 7$

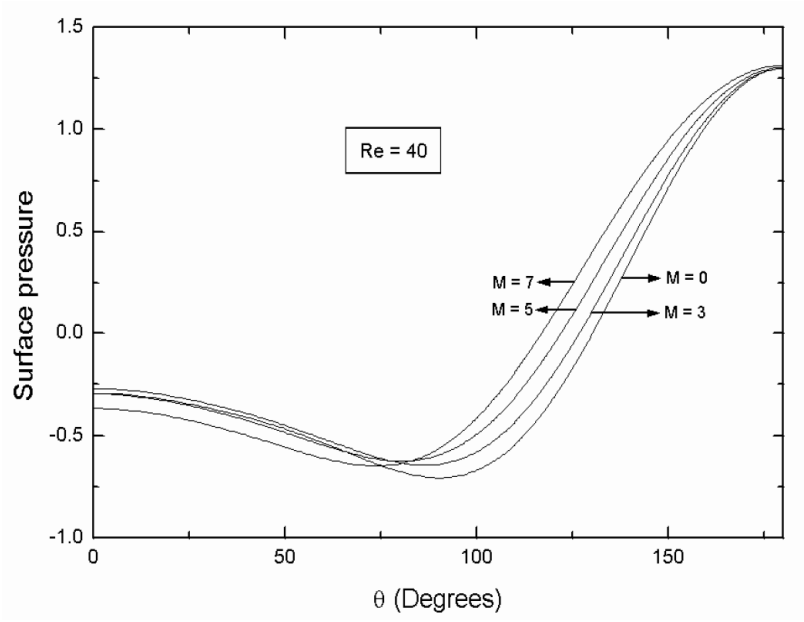


Figure 14. Surface pressure for $Re = 40$ at different M values

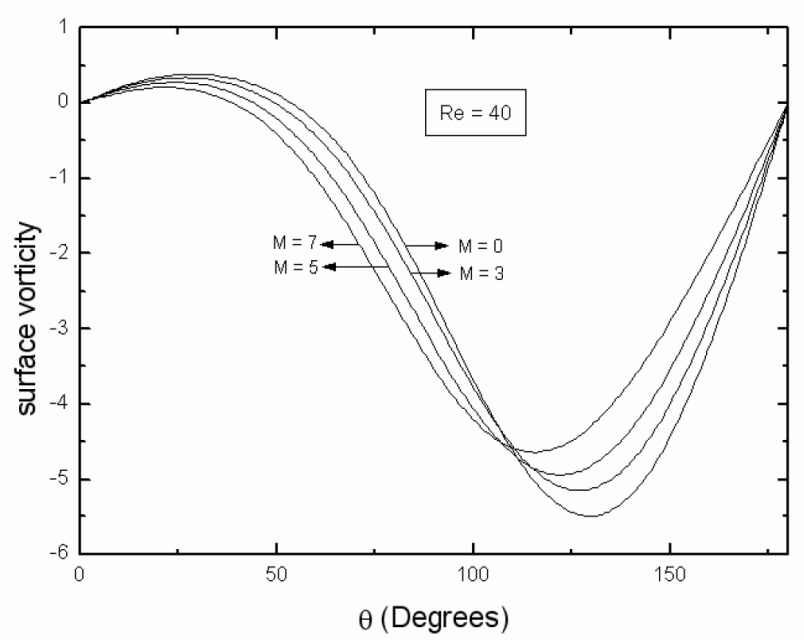


Figure 15. Surface vorticity for $Re = 40$ at different M values

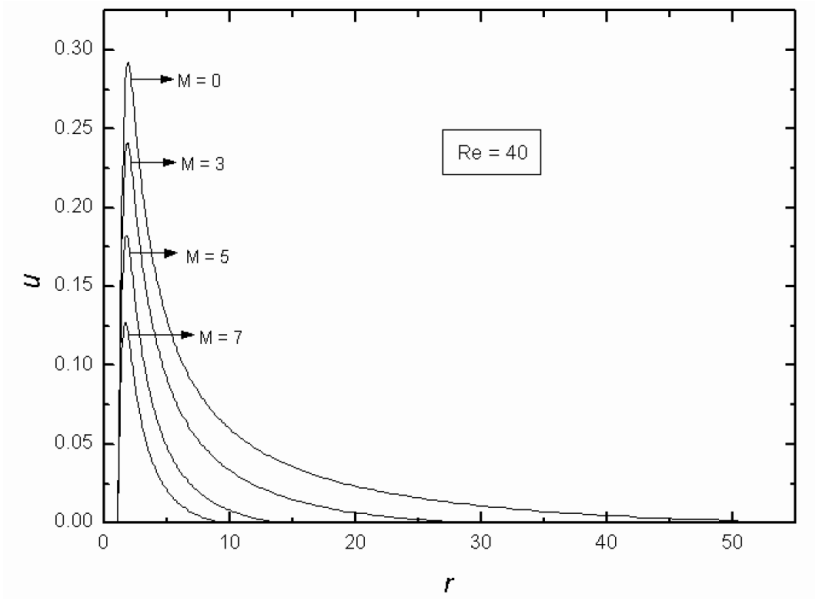


Figure 16. Radial component of velocity within boundary layer at $\theta = 90^\circ$

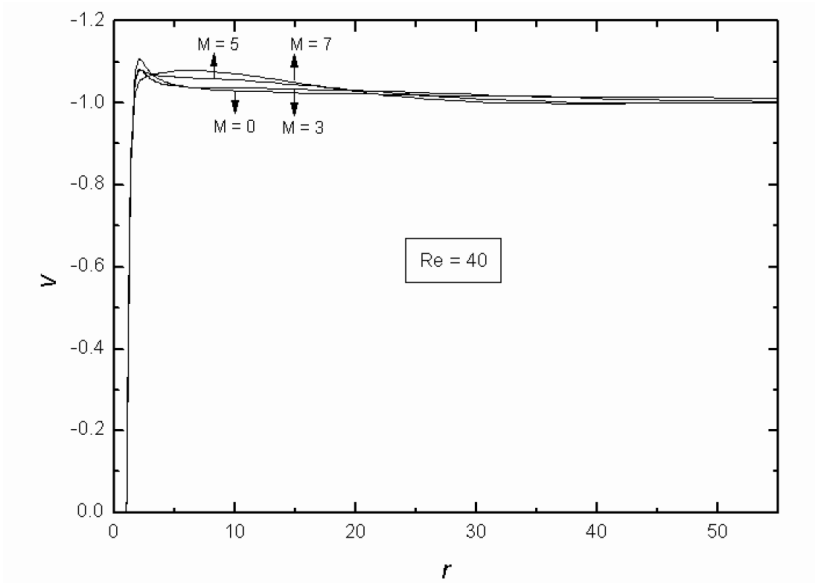


Figure 17. Transverse component of velocity within boundary layer at $\theta = 90^\circ$

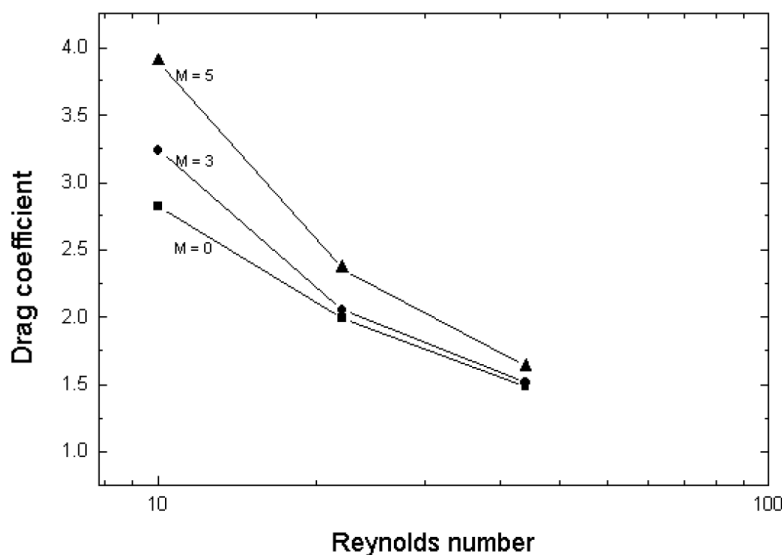


Figure 18. Drag coefficient versus Reynolds number at different M values

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