Journal of Computational and Applied Mechanics, Vol. 6., No. 1., (2005), pp. 15–25

SIMILARITY SOLUTION FOR MHD FLOW THROUGH VERTICAL POROUS PLATE WITH SUCTION

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[Received: October 28, 2004]

Abstract. In this paper a similarity analysis is made for the forced and free convection boundary layer flow in a semi-infinite expanse of an electrically conducting viscous incompressible fluid past a semi-infinite non-conducting porous plate with suction. A uniform magnetic field is applied normal to the plate. A time dependent suction is also introduced. The governing equations of the problems are then reduced to linear similarity equations, which are made local by introducing suitable similarity parameters. These local similarity equations are solved numerically by an adapting shooting method which uses the Nachtsheim-swigert interaction technique. Effects of various parameters on the velocity and temperature fields across the boundary layer are investigated. Numerical results for the velocity and temperature distributions are shown graphically.

Mathematical Subject Classification: 76W05
Keywords: magneto-hydrodynamics, forced and free convection, porous plate, magnetic field

1. Introduction

Magneto-hydrodynamics (MHD) is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. Many natural phenomena and engineering problems are worth being subjected to an MHD analysis. Magneto-hydrodynamic equations are ordinary electromagnetic and hydrodynamic equations modified to take into account the interaction between the motion of the fluid and the electromagnetic field. The formulation of the electromagnetic theory in a mathematical form is known as Maxwell's equation.

The effect of the gravity field is always present in forced flow heat transfer as a result of the buoyancy forces connected with the temperature differences. Usually they are of a small order of magnitude so that the external forces may be neglected.

There has recently been a considerable interest in the effect of body forces on forced convection phenomena. The effect of the gravity field is always present in forced flow heat transfer as a result of the buoyancy forces caused by temperature differences. Usually they are small and can be neglected. In certain engineering problems, however, they cannot be left out of consideration. It is important to realize that the heat transfer in mixed convection can be significantly different from that both in pure natural convection and in pure forced convection.

The study of forced and free convection flow and heat transfer for electrically conducting fluids past a semi-infinite porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as geophysics, astrophysics, boundary layer control in the field of aerodynamics (Soundalgekar et al. 1977 [1]). The physical model and geometrical coordinates are shown in Figure 1.

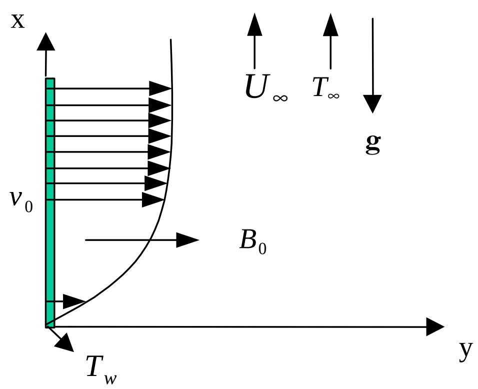

Figure 1. The physical model

In many practical fields, we found significant temperature differences between the surface of the hot body and the free stream. These temperature differences cause density gradients in the fluid medium and in presence of gravitational forced free convection effects become important. by applying transverse magnetic field Agrawal *et al.* [2] found that the rate of heat transfer from the plate to the fluid decreases as the suction velocity increases and the skin friction decreases with increasing Hartman number. Georgantopulos et al. [3], Raptis et al. [4, 5], Soundalgekar and Takher [1] and many others elucidated the various aspects of MHD free convection flows with suction. Some of the earlier works were done by Sparrow et al. [6], Lloyad and Sparrow [7], Wilks [8], Chen et al. [9], Tingwei et al. [10], and Raju et al.

[11]. In addition to the above, studies about convective flows in a porous medium have attracted considerable interest owing to their applications in geophysical and geothermal problems. Theoretical studies of such a flow under free convection were done among others by Bestmen [12, 13], Raptis [4, 5] and Perdikis [14]. Sattar [15] obtained an analytic solution of the free and forced convection flow through a porous medium near the leading edge by a perturbation method adopted by Singh and Dikshit [16]. Soundalgekar et al. [17], Perdiks [14], Sattar [18] made analytical studies on the combined forced and free convection flow in a porous medium. In these studies it has been generally recognized that $\gamma = G_r/R_e^2$ (where G_r is the Grashof number and R_e is the Reynolds number) is the governing parameter for a vertical plate. In the present work, therefore the effect of the large suction on the MHD forced and free convection flow past a vertical porous plate is studied. Solutions to the problem posed are found numerically for the whole range of the buoyancy parameter γ that is considered to be the driving force of the whole range of the combined forced and free convection.

2. **Governing equations of the flow and mathematical analysis**

Consider the forced and free convection flow of an incompressible viscous and electrically conducting fluid past a heated semi-infinite vertical porous plate.

The fluid is permeated by a strong magnetic field $\vec{B} = [0, B_0(x), 0]$. T_∞, U_∞ are the temperature and velocity of the uniform flow, respectively. The induced magnetic field is assumed to be negligible. This assumption is justified by the fact that the magnetic Reynolds number is very small. Further, since no external electric field is applied, the effect of polarization of the ionized fluid is negligible and it may also be assumed that the electric field $\vec{E} = 0$. Regarding the convection as a result of the effects of thermal diffusion, the equations of motion without Hall effects can be put into the following forms:

The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \,. \tag{1}$$

The momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\beta\,(T - T_\infty) + \frac{\sigma_0 B_0^2(x)}{\rho}\,(U_\infty - u) \ . \tag{2}$$

The energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \ . \tag{3}$$

The boundary conditions for the present problem are as follows

$$\begin{array}{llll} u = 0, & v = v_0(x), & T = T_w, & \text{if } \ y = 0 \ ; \\ u = U_\infty, & v = 0, & T = T_\infty, & \text{if } \ y \to \infty \ , \end{array} \tag{4}$$

where $v_0(x)$ is the velocity of suction and U_∞ is the free steam velocity.

Equations (1)-(3) constitute the basic equations which govern the physical problem considered here. Our next task is to make an approach that will lead to the solutions of these equations provided that the boundary conditions are given by equations (4).

In order to attain a similarity solution to our problem, the following transformations are applied:

$$lll\eta = y\sqrt{\frac{U_\infty}{2\upsilon x}}, \quad u = U_\infty f'(\eta), \quad f_w = v_0(x)\sqrt{\frac{2x}{\upsilon U_\infty}}\ , \qquad \theta = \frac{T - T_\infty}{T_W - T_\infty}\ , \tag{5}$$

where f_w is the transpiration parameter.

We now introduce the following dimensionless local parameters in the above equation:

$$G_{rx} = \frac{U_\infty g\beta\,(T_w - T_\infty)\,x^2}{\upsilon v_0^2\,(x)}\ ,\ R_{ex} = \frac{U_\infty x}{\upsilon}\quad , f_w = v_0(x)\sqrt{\frac{2x}{\upsilon U_\infty}}\ ,$$

$$\gamma = \frac{G_{rx}}{R_{ex}^2}\ ,\ M = \frac{\sigma_0 B_0^2\,(x)\,2x}{U_\infty \rho\, f_w^2}\ .$$

After performing the transformations we obtain the differential equations

$$f''' + ff'' + f_w^2 M(1 - f') = -\gamma f_w^2 \theta\,, \tag{6}$$

$$\theta'' + P_r f\theta' = 0\ . \tag{7}$$

Making use of the dimensionless variables (5) the boundary conditions (4) can be manipulated into form

$$\begin{array}{llll} f = f_w, & f' = 0, & \theta = 1 & \text{if } \eta = 0\,; \\ f' = 1, & & \theta = 0, & \text{if } \eta \to \infty\,. \end{array} \tag{8}$$

It is interesting to note that when suction is absent, i.e. $f_w = 0$, equation (6) reduces to the ordinary Blasius equation. The solutions of the Blasius equation are referred to as the Blasius solutions. They have also been studied by Schlichting [19].

On the other, hand if $\gamma \to 0$, R_e is large and the forced convection is dominating, equation (6) corresponds to the ordinary Falkner and Skan equation. In this case the boundary conditions differ largely from those of the original Falkner and Skan equation.

3. **Numerical scheme and procedure:**

Equations (6) and (7) with boundary conditions (8) are solved numerically using a standard initial value solver, i.e., the shooting method. For the purpose of this method, we applied the Nacthsheim-Swigert iteration technique (Nachtsheim & Swigert, 1965).

In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed and the differential equation is integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach, one naturally inquires whether or not there is a systematic way of finding each succeeding (assumed) value of the missing initial condition.

The boundary conditions (8) associated with the linear ordinary differential equations (6) and (7) of the boundary layer type are of the two-point asymptotic class. Two-point boundary conditions have values of the dependent variable specified at two different values of the independent variable. Specification of an asymptotic boundary condition implies the value of velocity approaches to unity and the value of temperature approaches to zero as the outer specified value of the independent variable is approached. The method of numerically integrating two-point asymptotic boundary value problem of the boundary layer type, the initial value method, requires that the problem be recast as an initial value problem. Thus it is necessary to set up as many boundary conditions at the surface as there are at infinity. The governing differential equations are then integrated with these assumed surface boundary conditions. If the required outer boundary condition is satisfied, a solution has beeen achieved. However, this is not generally the case. Hence a method must be devised to logically estimate the new surface boundary conditions for the next trial integration. Asymptotic boundary value problems such as those governing the boundary layer equations are further complicated by the fact that the outer boundary condition is specified at infinity. In the trial integration infinity is numerically approximated by some large value of the independent variable. There is no a priori general method of estimating this value. Selection of too small a maximum value for the independent variable may not allow the solution to asymptotically converge to the required accuracy. Selecting a large value may result in divergence of the trial integration or in slow convergence of surface boundary conditions required satisfying the asymptotic outer boundary condition. Selecting too large a value of the independent variable is expensive in terms of computer time. Nachtsheim-Swigert developed an iteration method, which overcomes these difficulties. Extension of the Nachtsheim-Swigert iteration shell to above equation system of differential equations (6) and (7) is straightforward. In equation (8) there are two asymptotic boundary conditions and hence two unknown surface conditions $f'(0)$ and $\theta'(0)$.

4. **Results and discussion**

In this paper we have attempted to solve the combined free and forced convection flow in a semi-infinite vertical porous plate with suction. Locally similar solutions of this problem have been obtained by introducing a similarity parameter taken to be a time dependent scale. The suction velocity is taken to be a function of time. Under these conditions the solutions to the problem are finally obtained by employing a numerical technique.

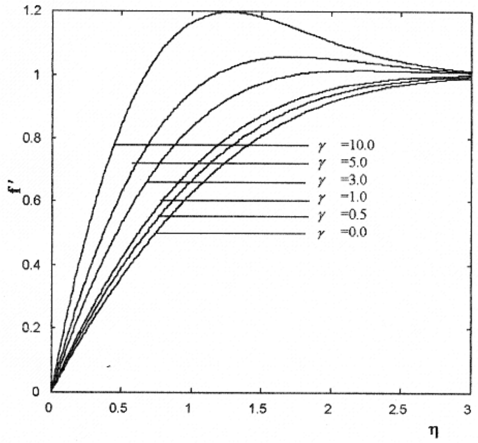

Figure 2. Dimensionless velocity for different values of γ and $P_r = 0.71,\ f_w = 0.5,\ M = 2.0$

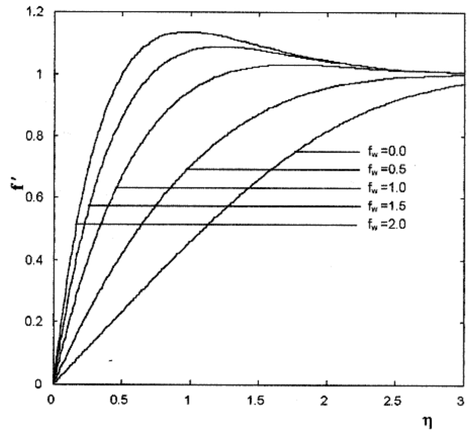

Figure 3. Dimensionless velocity for different values of f_w and $P_r = 0.71,\ \gamma = 1.0,\ M = 2.0$

For the purpose of discussing the numerical solutions, the effects of various parameters on the flow behavior have been determined for different values of the buoyancy parameter γ, suction/ injection parameter f_w, Prandtl number P_r and magnetic parameter M. Since there are four parameters of interest in the present problem, which can be varied, we have focused attention on the values $\gamma = 0.0,\ 0.5,\ 1.0,\ 3.0,\ 5.0,\ 10.0$; $f_w = 0.0,\ 0.5,\ 1.0,\ 1.5,\ 2.0$; $P_r = 0.71,\ 1.0,\ 7.0$ and $M = 2,\ 4, 6$.

In Figure 2, the effects of the driving parameter γ on the velocity profiles are shown. It is obvious from this Figure that the velocity increases with the increasing values of γ, which signifies that the velocity is higher in the case of pure free convection than

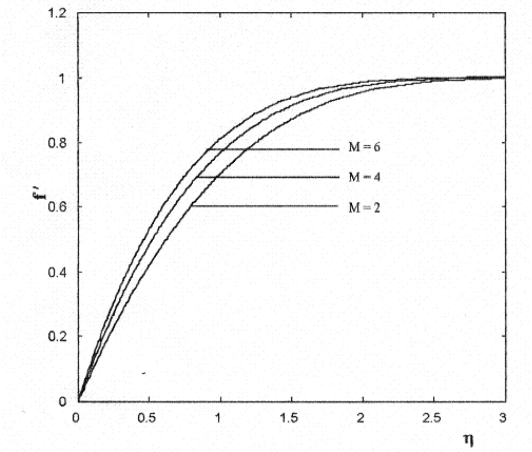

Figure 4. Dimensionless velocity for different values of M and $P_r = 0.71,\ \gamma = 1.0,\ f_w = 0.5$

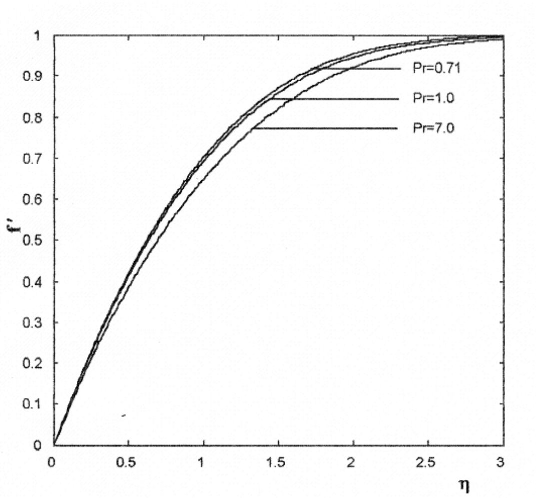

Figure 5. Dimensionless velocity for different values of P_r and $M = 2.0,\ \gamma = 1.0,\ f_w = 0.5$

for pure forced convection. Moreover, in the case of pure free convection, the *velocity* is found to overshoot.

In the case of mixed convection ($\gamma = 1$) a rise in f_w (suction) causes a rise in the velocity as shown in Figure 3. As is shown in the Figure, if suction increases, there is a decrease in the boundary layer growth, which indicates that suction destabilizes the boundary layer.

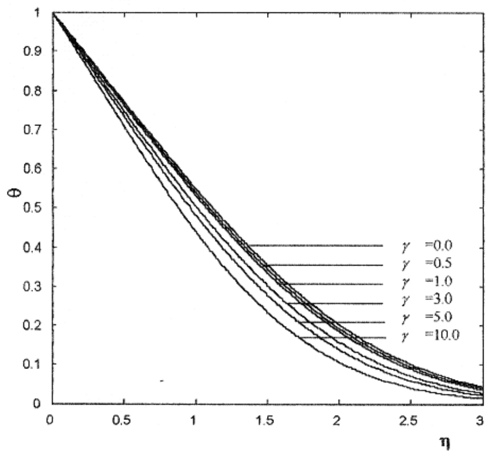

Figure 6. Dimensionless temperature for different values of γ and $M = 2.0,\ P_r = 0.71,\ f_w = 0.5$

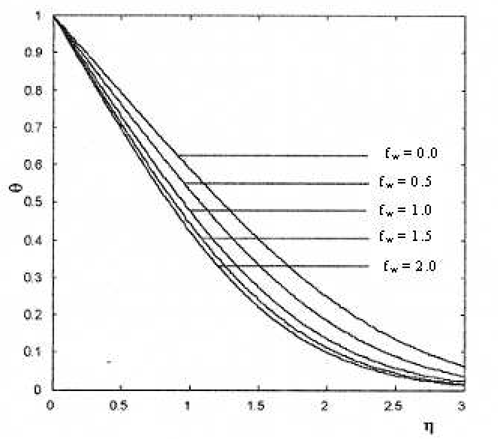

Figure 7. Dimensionless temperature for different values of f_w and $M = 2.0,\ P_r = 0.71,\ \gamma = 1.0$

The effects of the magnetic parameter on the velocity profiles are displayed in Figure 4, which shows that the velocity increases with the increase of the magnetic parameter.

In Figure 5 the effects of the Prandtl number on the velocity profiles are shown. It can be seen from this Figure that the velocity profiles decrease due to the increasing values of the Prandtl number.

In Figure 6, the effects of the buoyancy parameter γ on the temperature profiles are shown. From this Figure it can be seen that the temperature decreases with the increase of γ.

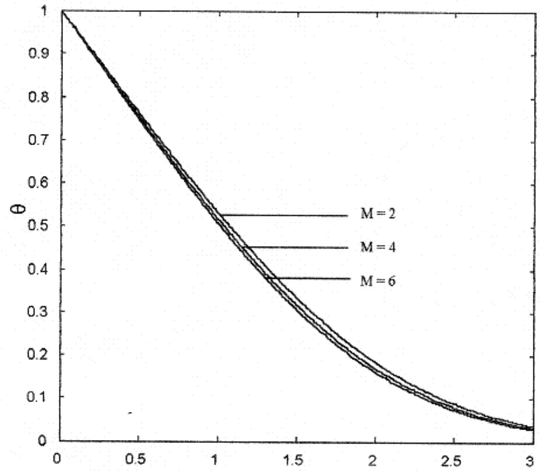

Figure 8. Dimensionless temperature for different values of M and $P_r = 0.71,\ \gamma = 1.0,\ f_w = 0.5$

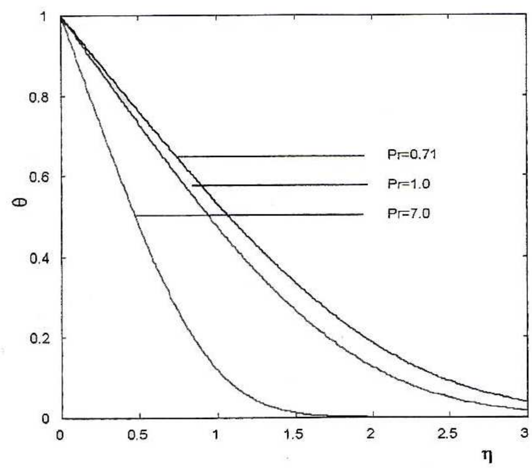

Figure 9. Dimensionless temperature for different values of P_r and $\gamma = 1.0,\ f_w = 0.5,\ M = 2.0$

In Figure 7, the effects of the suction parameter on the temperature profiles are shown. From this Figure it can be seen that the temperature decreases with the increase of the suction parameter.

In Figure 8, the effects of the magnetic parameter on the temperature profiles are shown. From this Figure it can be seen that the temperature decreases with the increase of magnetic parameter.

The effects of the Prandtl number on the temperature are depicted in Figure 9. From this Figure it can be noted that the temperature profiles decrease with the increase in the Prandtl number. These effects are the same as those for velocity profiles.

5. Conclusion

We have examined the governing equations for an unsteady incompressible fluid past a semi-infinite vertical porous plate embedded in a porous medium and subjected to the presence of a transverse magnetic field. Numerical results are presented to illustrate the details of the various parameters. The values of driving parameter γ as shown above, however, correspond to three regimes, namely the predominantly forced convection regime, the mixed convection regime and the predominantly free convection regime. For γ=0,the gravity induced free convection is absent and the flow is completely forced over the surface. For low values of γ $(0 < \gamma < 1)$, the forced convection dominates and the local similarity solutions are the same as those in the case of forced convection only, which was studied by Narain and Uberoi [20]. The large values of $\gamma \gg 1)$ are interesting from a physical point of view. For this purpose, value of $\gamma = 5$ can be essentially treated as the free convection representation.

Appendix A. Nomenclature

x, y, z	cartesian coordinates	g	gravitational acceleration
t	time	G_r	Grashof number
f_w	transpiration parameter	C_p	specific heat
u, v	fluid velocities	R_{ex}	Reynolds number
V_0	velocity of suction	T	temperature
μ	kinematics viscosity	γ	buoyancy parameter
η	similarity variable	T_w	plate temperature
υ	coefficient of kinematics viscosity	T_∞	free steam temperature
θ	dimensionless temperature	T_0	reference temperature
ρ	fluid density	κ	heat diffusivity coefficient
$\overrightarrow{B}$	magnetic field	U_∞	free steam velocity
β	coefficient of volume expansion	$\eta = y\sqrt{\frac{U_\infty}{2\upsilon x}}$	similarity variable
$\overrightarrow{E}$	electric field	$\theta = \frac{T-T_\infty}{T_w-T_\infty}$	dimensionless temperature

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