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HOMOTOPY PERTURBATION METHOD FOR ANALYSIS OF SQUEEZING AXISYMMETRIC FLOW OF NEWTONIAN FLUID UNDER THE EFFECTS OF SLIP AND MAGNETIC FIELD

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Abstract. In this paper, a steady two-dimensional axisymmetric flow of an incompressible viscous fluid under the influence of a uniform transverse magnetic field with slip boundary condition is studied. Using suitable similarity variables, the developed nonlinear partial differential equation of the flow phenomena is converted to a nonlinear ordinary differential equation which is solved analytically using homotopy perturbation method. By comparing the results of approximate analytical methods in this work with the results of numerical method using Runge-Kutta coupled with shooting method, the verification and the accuracy of approximate analytical solution are established. Thereafter, through the developed analytical solutions, the effects of pertinent flow, magnetic field and slip parameters on the steady two-dimensional axisymmetric flow of the viscous fluid are investigated, graphically illustrated and discussed. It is observed from the results that the velocity of the fluid increases with increase in the magnetic parameter under slip condition while the velocity decreases with increase in the magnetic field parameter under the no slip condition. By increasing the slip parameter, the velocity of the fluid increases and the fluid velocity decrease as the Reynolds number increases. It is hoped that this study will enhance and advance the understanding of axisymmetric squeezing flow of viscous fluid under no-slip and slip conditions.

Mathematical Subject Classification: 05C38, 15A15
Keywords: First-grade fluid, squeezing flow, magnetic field, slip boundary, homotopy perturbation method

1. Introduction

The analysis of squeezing flow between two parallel surfaces has received considerable and appreciable research attention as its applications increase continuously in various industrial and engineering processes. The numerous applications of such flow processes is evident in industrial and engineering applications such as moving pistons, chocolate fillers, hydraulic lifts, electric motors, flow inside syringes and nasogastric tubes, compression, and injection, power transmission squeezed film and polymers processing. In such applications, the flow of fluid is performed as a result of the moving apart or the coming together of two parallel plates. Following the pioneer work and the basic formulations of squeezing flows under lubrication assumption by Stefan

[1], there have been increasing research interests and many scientific studies on these types of flow. In past work Reynolds [2] analyzed the squeezing flow between elliptic plates while Archibald [3] investigated the same problem for rectangular plates. Earlier studies on squeezing flows were based on the Reynolds equation whose insufficiency for some cases has been shown by Jackson [4] and Usha and Sridharan [5]. Therefore, there have been several attempts and renewed research interests by different researchers to properly analyze and understand squeezing flows [6–15]. In the past efforts to analyze such flow process, Rashidi et al. [16] used the homotopy analysis method (HAM) to develop analytical approximate solutions to study the unsteady two-dimensional axisymmetric squeezing flow between parallel plates while Duwairi et al. [17] investigated effects of squeezing on heat transfer of a viscous fluid between parallel plates. Qayyum et al. [18] studied the squeezing flow of non-Newtonian second grade fluids and micro-polar models, presenting the effect on velocity profiles. Hamdan [13] analyzed the effect of squeezing flow on dusty fluids discussing squeeze effect on fluid flow. Mahmood et al. [19] investigated the effects of Prandtl's number and Nusselt number on the squeezed flow and heat transfer over a porous surface for viscous fluids. Hatami and Jing [20] applied the differential transformation method to study the natural convection of a non–Newtonian nanofluid between two vertical plates and Newtonian nanofluid between horizontal plates. Mohyud-Din et al. [21] investigated heat and mass transfer for the flow of a nanofluid between rotating parallel plates while Mohyud-Din and Khan [22] analyzed the nonlinear radiation effects on squeezing flow of a Casson fluid between parallel disks. Qayyum et al. [23] modeled and applied the homotopy perturbation method to analyze the unsteady axisymmetric squeezing fluid flow through the porous medium channel with slip boundary. Qayyum and Khan [24] presented the behavioral study of unsteady squeezing flow through porous medium using the homotopy perturbation method. Mustafa et al. [25] presented their study on the heat and mass transfer in unsteady fluid flow under squeezed flow between two parallel plates using the homotopy analysis method.

In order to study the influence of magnefield on the sqeezing flow of non-Newtonian fluid, Siddiqui et al. [26] adopted the homotopy perturbation method to the magnetic effect of squeezing viscous magnetohydrodynamics (MHD) fluid flow. A few years later, Domairry and Aziz [27] used the homotopy perturbation method (HPM) to study the MHD squeezed flow between two parallel disks with suction or injection. Also, the effect of squeeze on copper-water and copper-kerosene nanofluid between two parallel plates subjected to magnetic field was studied by Acharya et al. [28] using the differential transformation method (DTM). Ahmed et al. [29] analyzed magneto hydrodynamic (MHD) squeezing flow of a Casson fluid between parallel disks. A year later, Ahmed et al. [30] investigated MHD flow of an incompressible fluid through porous medium between dilating and squeezing permeable walls. The same year, Khan et al. [31] studied unsteady two-dimensional and axisymmetric squeezing flow between parallel plates. The same authors [32] studied MHD squeezing flow between two infinite plates while Hayat et al. [33] had earlier investigated the effect of squeezing flow of second grade fluid between two parallel disks. Khan et al. [34] analyzed unsteady squeezing flow of Casson fluid with magnetohydrodynamic effect and passing through porous medium while Ullah et al. [35] used homotopy perturbation method to present an analytical solution of squeezing flow in porous medium with MHD effect. Thin Newtonian liquid films squeezing between two plates were studied by Grimm [10]. Squeezing flow under the influence of magnetic field is widely applied to bearing with liquid-metal lubrication [36–39].

Islam et al [40] studied squeezing fluid flow between the two infinite parallel plates in a porous medium channel. In case of many polymeric liquids when the weight of molecule is high, they show slip at the boundary. The no-slip boundary condition is not applicable in this case. In many cases such as thin film problems, rarefied fluid problems, fluids containing concentrated suspensions, and flow on multiple interfaces, the no-slip boundary condition fails to work. Navier [41], for the first time, proposed the general boundary condition which demonstrates the fluid slip at the surface. The difference of fluid velocity and velocity of the boundary is proportional to the shear stress at that boundary. The proportionality constant is named the slip parameter with length as its dimension. The slip condition is of great importance especially when fluids with elastic character are under consideration [42]. Newtonian fluid was considered by Ebaid [43] to study the effects of magnetic field and wall slip conditions on the peristaltic transport in an asymmetric channel. This has great importance in medical sciences, particularly in polishing artificial heart valves and internal cavities in many manufactured parts achieved by embedding such fluids with abrasives [44]. The influence of slip on the peristaltic motion of third-order fluid in asymmetric channel is studied by Hayat et al. [45]. The effects of slip condition on the rotating flow of a third-grade fluid in a nonporous medium are investigated by Hayat and Abelman [46]. Their work was extended to a porous medium and obtaining the numerical solutions for the steady magnetohydrodynamics flow of a third grade fluid in a rotating frame is presented by Abelman et al. [47]. Ullah et al. [48] presented an approximation of first-grade MHD squeezing fluid flow with slip boundary condition using differential transformation method and optimal homotopy asymptotic method.

The past efforts in analyzing the squeezing flow problems have been largely based on the applications of approximate analytical methods such as the homotopy analysis method, differential transformation method, domian decomposition method, variation iteration method, optimal homotopy asymptotic method etc. In this paper, magnetohydrodynamic squeezing flow of first-grade fluid with slip boundary condition between two infinite plates is analyzed using the homotopy perturbation method. The study is carried out to further study and analyze the applications and limitations of the homotopy perturbation method to the fluid flow problem. Also, effects of pertinent flow, magnetic field and slip parameters are studied. By comparing the results of approximate analytical methods in this work with the numerical method using Runge-Kutta coupled with the shooting method, the verification and the accuracy of this approximate analytical solution is established.

2. Problem formulation

Consider a squeezing flow of an incompressible Newtonian fluid with constant density ρ and viscosity μ, squeezed between two large planar parallel plates separated

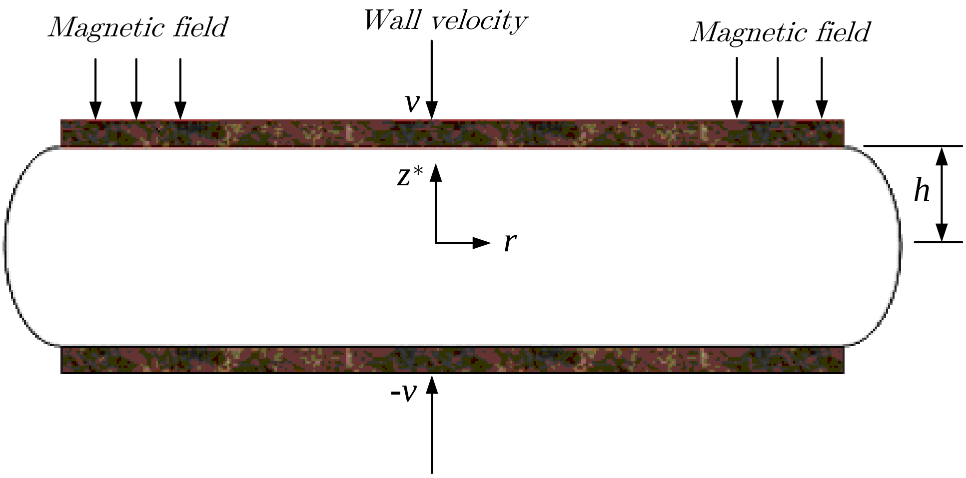

Figure 1. Model of the squeezing flow of viscous fluid under transverse uniform magnetic field

by a small distance $2h$ approaching each other with a low constant wall velocity v in the presence of a magnetic field, as shown in Figure 1. Assume that the flow is quasi steady. Then the Navier-Stokes equations governing such flow if the inertial terms are retained are:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\nabla \cdot \mathbf{v})\mathbf{v} \right] = \nabla \cdot \mathbf{T} + (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} \tag{2}$$

where ∇ denotes the gradient operator, $\mathbf{v}$ is the fluid velocity, ρ is the fluid density, $\mathbf{T} = \mu\mathbf{A} - p\mathbf{I}$ is the Cauchy stress tensor, p is the fluid pressure $\mathbf{I}$ is the identity tensor μ the fluid dynamic viscosity, $\mathbf{A} = \frac{1}{2}(\mathbf{v}\nabla + \nabla\mathbf{v})$, $\mathbf{B} = \mathbf{B}_o + b\mathbf{B}_o$ is the total magnetic field given in terms of the the magnetic field intensity $\mathbf{B}_o$ and the imposed and induced magnetic fields parameter b. The modified Ohm's law and Maxwell's equations. in the absence of displacement currents, are:

$$\mathbf{J} = \sigma \left[\mathbf{E} + \mathbf{v} \times \mathbf{B} \right], \quad \nabla \cdot \mathbf{B} = 0, \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_m \mathbf{J}, \quad \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}\,. \tag{4}$$

Here $\mathbf{J}$ is the electric current density, σ represents the electrical conductivity, $\mathbf{E}$ is the electric field, and μ_m is the magnetic permeability. If ρ and μ_m are constants, b is negligible as compared to $\mathbf{B}_o$, $\mathbf{B}$ is perpendicular to $\mathbf{v}$ so that the Reynolds number is small with no electric field in the fluid flow region and then the magneto hydrodynamic force involved can be written as

$$\mathbf{J} \times \mathbf{B} = -\sigma \mathbf{B}_o^2 \mathbf{v}\,. \tag{5}$$

Assuming that the plates are non-conducting and the magnetic field is applied along the z-axis the gap distance $2h$ between the plates changes slowly with time t for small values of the velocity $\mathbf{v}$ so that it can be taken as constant. The flow is axisymmetric and is investigated in the cylindrical coordinates (r, θ, z^*) chosen in such a way that the z^*-axis is perpendicular plates for which $z^* = \pm h$ are the vertical coordinates. It is clear that $\mathbf{v}$ can now be given in the form $\mathbf{v} = (v_r, 0, v_z)$. If the body forces are negligible – this is an assumption – the Navier-Stokes equations in cylindrical coordinates are [1, 6, 7]:

$$\frac{\partial p}{\partial r} - \rho\Omega v_r = -\mu\frac{\partial \Omega}{\partial z^*} - \sigma B_o^2 \mathrm{v}_r\,, \tag{6}$$

$$\frac{\partial p}{\partial z^*} - \rho\Omega v_r = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\Omega\right)\,, \tag{7}$$

where

$$\Omega\left(r, z^*\right) = \frac{\partial \mathrm{v}_z}{\partial r} - \frac{\partial \mathrm{v}_r}{\partial z^*}\,. \tag{8}$$

Introducing the stream function $\psi(r, z^*)$ we get

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial z^*}\,, \quad v_z = -\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{9a}$$

and a generalized pressure for the cylindrical coordinate system as follows

$$p = \frac{\rho}{2}\left(v_r^2 + v_z^2\right)\,. \tag{9b}$$

Eliminating the pressure term from equations (6) and (7) we have

$$\rho\left[\frac{\partial\left(\psi, E^2\psi/r^2\right)}{\partial\left(r, z^*\right)}\right] = -\frac{\mu}{r}E^2\psi + \frac{\sigma B_0^2}{r}\frac{\partial^2\psi}{\partial z^{*2}} \tag{10}$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^{*2}}\,. \tag{11}$$

Using the transformation $\psi\left(r, z^*\right) = r^2 f\left(z^*\right)$ equation (10) can be written as

$$f^{(4)}\left(z^*\right) - \frac{\sigma B_o^2}{\mu}f^{(2)}\left(z^*\right) + 2\frac{\rho}{\mu}f\left(z^*\right)f^{(3)}\left(z^*\right) = 0\,. \tag{12}$$

This equation is associated with the following slip boundary conditions:

$$\begin{aligned} &f\left(0\right) = 0\,, && f^{(2)}\left(0\right) = 0\,,\\ &f\left(h\right) = \frac{v_z}{2}\,, && f^{(1)}\left(h\right) = \beta f^{(2)}\left(h\right) \end{aligned} \tag{13}$$

where β is the slip parameter.
Applying the following dimensionless variables

$$F = \frac{2f}{v_z}\,, \quad z = \frac{z^*}{h}\,, \quad R = \frac{\rho h v_z}{\mu} \quad \text{and} \quad m = B_o h\sqrt{\frac{\sigma}{\mu}}\,, \tag{14}$$

equation (12) can be manipulated into the following form

$$F^{(4)}\left(z\right) - m^2F^{(2)}\left(z\right) + RF\left(z\right)F^{(3)}\left(z\right) = 0. \tag{15}$$

The boundary conditions can be obtained in the same way from (13):

$$\begin{aligned} &F\left(0\right)=0\,, \qquad F^{(2)}\left(0\right)=0\,,\\ &f\left(1\right)=1\,, \qquad F^{(1)}\left(1\right)=\gamma F^{(2)}\left(1\right)\,. \end{aligned} \tag{16}$$

Here $\gamma = \beta/h$ while R and m are the Reynolds and Hartmann numbers respectively. It should be noted that γ is a dimensionless slip parameter.

3. Problem formulation

3.1. **The basic idea of homotopy perturbation method.** It is very difficult to develop a closed-form solution for the above non-linear equation (15). Therefore, recourse has to be made to either the approximation analytical method, semi-numerical method or numerical method of solution. In this work, the homotopy perturbation method is used to solve the equation. In order to establish the basic idea for the homotopy perturbation method, consider a system of nonlinear differential equations given as

$$A\left(U\right)-f\left(r\right)=0, \quad r\in\Omega, \tag{17}$$

with the boundary conditions

$$B\left(u,\frac{\partial u}{\partial \eta}\right)=0, \qquad r\in\Gamma \tag{18}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω.

The operator A can be divided into two parts denoted by L and N where L is a linear operator and N is a non-linear operator. Equation (17) can therefore be rewritten as follows

$$L\left(u\right)+N\left(u\right)-f\left(r\right)=0\,. \tag{19}$$

By the homotopy technique, a homotopy $U\left(r,p\right):\Omega\times[0,1]\to R$ can be constructed, which satisfies the equation

$$H\left(U,p\right)=\left(1-p\right)\left[L\left(U\right)-L\left(U_o\right)\right]+p\left[A\left(U\right)-f\left(r\right)\right]=0, \quad p\in\left[0,1\right], \tag{20}$$

or the equation

$$H\left(U,p\right)=L\left(U\right)-L\left(U_o\right)+pL\left(U_o\right)+p\left[N\left(U\right)-f\left(r\right)\right]=0\,. \tag{21}$$

In equations (20) and (21) $p\in[0,1]$ is an embedding parameter and U_o is an initial approximation of U in (17) which should satisfy the boundary conditions.

Also, from equations (20) and (21) we will have

$$H\left(U,0\right)=L\left(U\right)-L\left(U_o\right)=0, \tag{22}$$

$$H\left(U,0\right)=A\left(U\right)-f\left(r\right)=0. \tag{23}$$

The changing process of p from zero to unity is just that of $U(r,p)$ from $U_o(r)$ to $U(R)$. This is referred to homotopy in topology. Using the embedding parameter p as a small one, solution of equations (20) and (21) can be represented by a power series in p as is shown in equation (24):

$$U=U_o+pU_1+p^2U_2+\cdots \tag{24}$$

It should be pointed out that the values of p are between 0 and 1; $p = 1$ produces the best result. Therefore, setting P to 1 results in the approximative solution of equation (17):

$$u = \lim_{p \to 1} U = U_o + U_1 + U_2 + \cdots \tag{25}$$

The basic idea expressed above is a combination of the homotopy and perturbation methods. Hence, the method is called homotopy perturbation method (HPM), designed to the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantages of the traditional perturbation techniques. The series in equation (25) is convergent for most cases.

3.2. **Application of the homotopy perturbation method to the present problem.** According to the homotopy perturbation method (HPM), one can construct a homotopy for equation (15) as

$$H(z,p) = (1-p)\,\tilde{F}^{(4)} + p\left[\tilde{F}^{(4)} - m^2\tilde{F}^{(2)} + R\tilde{F}\tilde{F}^{(3)}\right] \tag{26}$$

Using the embedding parameter p as a small parameter, the approximative solution of equation (15) can be represented in the form of a power series in p as is given by equation (24):

$$\tilde{F} = \tilde{F}_o + p\tilde{F}_1 + p^2\tilde{F}_2 + p^3\tilde{F}_3 + \cdots \tag{27}$$

Upon substituting (27) into equation (26), expanding the equation obtained and collecting then all terms with the same order together, the resulting equation appears in the form of a polynomial in p. After equating each coefficient of the polynomial in p to zero, we arrive at a set of differential equations and the corresponding boundary conditions as:

$$\begin{aligned}
p^0:\ & \tilde{F}_0^{(4)} = 0,\\
& \tilde{F}_0(0) = 0,\ \ \tilde{F}_0^{(2)}(0) = 0,\ \ \tilde{F}_0(1) = 1,\ \ \tilde{F}_0^{(1)}(1) = \gamma\tilde{F}_0^{(2)}(1);
\end{aligned} \tag{28}$$

$$\begin{aligned}
p^1:\ & \tilde{F}_1^{(4)} - m^2\tilde{F}_0^{(2)} + R\tilde{F}_0\tilde{F}_0^{(3)} = 0,\\
& \tilde{F}_1(0) = 0,\ \ \tilde{F}_1^{(2)}(0) = 0,\ \ \tilde{F}_1(1) = 1,\ \ \tilde{F}_1^{(1)}(1) = \gamma\tilde{F}_1^{(2)}(1);
\end{aligned} \tag{29}$$

$$\begin{aligned}
p^2:\ & \tilde{F}_2^{(4)} - m^2\tilde{F}_1^{(2)} + R\tilde{F}_1\tilde{F}_0^{(3)} + R\tilde{F}_0\tilde{F}_1^{(3)} = 0,\\
& \tilde{F}_2(0) = 0,\ \ \tilde{F}_2^{(2)}(0) = 0,\ \ \tilde{F}_2(1) = 1,\ \ \tilde{F}_2^{(1)}(1) = \gamma\tilde{F}_1^{(2)}(1);
\end{aligned} \tag{30}$$

$$\begin{aligned}
p^3:\ & \tilde{F}_3^{(4)} - m^2\tilde{F}_2^{(2)} + R\tilde{F}_2\tilde{F}_0^{(3)} + R\tilde{F}_1\tilde{F}_1^{(3)} + R\tilde{F}_0\tilde{F}_2^{(3)} = 0,\\
& \tilde{F}_3(0) = 0,\ \ \tilde{F}_3^{(2)}(0) = 0,\ \ \tilde{F}_3(1) = 1,\ \ \tilde{F}_3^{(1)}(1) = \gamma\tilde{F}_1^{(2)}(1);
\end{aligned} \tag{31}$$

$$\begin{aligned}
p^4:\ & \tilde{F}_4^{(4)} - m^2\tilde{F}_3^{(2)} + R\tilde{F}_3\tilde{F}_0^{(3)} + R\tilde{F}_3\tilde{F}_0^{(3)} + R\tilde{F}_2\tilde{F}_1^{(3)} + R\tilde{F}_1\tilde{F}_2^{(3)} + R\tilde{F}_0\tilde{F}_3^{(3)} = 0\\
& \tilde{F}_4(0) = 0,\ \ \tilde{F}_4^{(2)}(0) = 0,\ \ \tilde{F}_4(1) = 1,\ \ \tilde{F}_4^{(1)}(1) = \gamma\tilde{F}_1^{(2)}(1);
\end{aligned} \tag{32}$$

$$\begin{aligned}
p^5:\ & \tilde{F}_5^{(4)} - m^2\tilde{F}_4^{(2)} + R\tilde{F}_3\tilde{F}_0^{(3)} + R\tilde{F}_4\tilde{F}_0^{(3)} + R\tilde{F}_3\tilde{F}_1^{(3)} + R\tilde{F}_2\tilde{F}_2^{(3)} +\\
& \qquad\qquad + R\tilde{F}_1\tilde{F}_3^{(3)} + R\tilde{F}_0\tilde{F}_4^{(3)} = 0\\
& \tilde{F}_5(0) = 0,\ \ \tilde{F}_5^{(2)}(0) = 0,\ \ \tilde{F}_5(1) = 1,\ \ \tilde{F}_5^{(1)}(1) = \gamma\tilde{F}_1^{(2)}(1).
\end{aligned} \tag{33}$$

After solving equations (28)-(33) we have

$$\tilde{F}_0(z) = \frac{3(2\gamma-1)z+z^3}{2(3\gamma-1)}, \tag{34}$$

$$\begin{aligned}\tilde{F}_1(z) = \frac{3R}{2(3\gamma-1)^2}z^7 + \left\{\frac{3m^2}{3\gamma-1}+\frac{9R(2\gamma-1)}{2(3\gamma-1)^2}\right\}z^5- \\ -\frac{1}{3(2\gamma+1)}\left\{\gamma\left(\frac{60m^2}{3\gamma-1}+\frac{90R(2\gamma-1)}{(3\gamma-1)^2}-\frac{63R}{(3\gamma-1)^2}\right)- \right.\\ \left. -\frac{12m^2}{3\gamma-1}-\frac{36R(2\gamma-1)}{(3\gamma-1)^2}-\frac{9R}{(3\gamma-1)^2}\right\}z^3- \\ -\left\{\frac{3m^2}{3\gamma-1}+\frac{9R(2\gamma-1)}{(3\gamma-1)^2}+\frac{3R}{2(3\gamma-1)^2}\right\}z.\end{aligned} \tag{35}$$

In the same manner, expressions such as $\tilde{F}_2(z)$, $\tilde{F}_3(z)$, $\tilde{F}_4(z)$, $\tilde{F}_5(z)$, $\tilde{F}_6(z)$ can be obtained. However, they are too complicated expressions to be included in this paper.

Setting p to 1 results in the approximative solution of equation (15):

$$F(z) = \lim_{p\to 1}\tilde{F}(z) = \tilde{F}_o(z)+\tilde{F}_1(z)+\tilde{F}_2(z)+\tilde{F}_3(z)+\tilde{F}_4(z)+\cdots \tag{36}$$

4. Results and discussion

The above analysis shows the development of approximate analytical methods of differential transformation and homotopy perturbation methods for the analysis of a quasi steady two-dimensional axisymmetric flow of an incompressible viscous fluid under the influence of a uniform transverse magnetic field with slip boundary condition. Using HPM, a series solution (10 terms) is obtained as it provides excellent approximation to the solution of the non-linear equation with good accuracy. Although the other approximative analytical methods such as differential transformation method (DTM), homotopy analysis method (HAM), Adomiam decomposition method (ADM) and variational iterative method (VIM) might likely present more accurate results than HPM in some nonlinear problems, the search for included unknown parameter(s) that will satisfy the end boundary condition(s) leads to additional computational cost in the generation of the analytical solutions to the problems using the approximate analytical methods (DTM, HAM, ADM, VIM, etc). Moreover, they have their own operational restrictions that severely narrow their functioning domain as they are limited to a small domain. Using DTM, HAM, ADM, VIM for large or infinite domains is done with either the application of before-treatment techniques such as domain transformation techniques, domain truncation techniques and conversion of the boundary value problems to initial value problems or the use of after-treatment techniques such as Pade-approximants, basis functions, cosine after-treatment technique, sine after-treatment technique and domain decomposition technique. This is because they were initially established for initial value problems. Amending the methods to boundary value problems especially for large or infinite domains boundary value problems leads us to search for unknown parameter(s) that will satisfy the end boundary condition (s). This drawback in the other approximation analytical methods is not experienced in HPM as such tasks of before- and after-treatment techniques are not necessarily required in HPM as it easily applied to the boundary value problems without any included unknown parameter in the solution as found in DTM, HAM, ADM, VIM.

Table 1. Comparison of results

$F(z)$		
z	NM	HPM
0.00	0.000000	0.000000
0.10	0.075739	0.075738
0.20	0.152935	0.152935
0.30	0.233046	0.233045
0.40	0.317540	0.317540
0.50	0.407893	0.407892
0.60	0.505591	0.505592
0.70	0.612134	0.612134
0.80	0.729034	0.729035
0.90	0.857813	0.857813
1.00	1.000000	1.000000

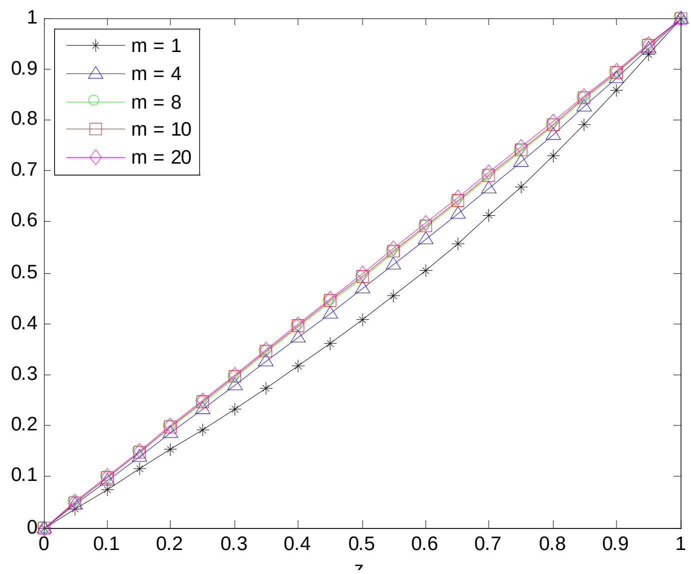

Figure 2. Effects of magnetic parameter on the flow behavior of the fluid under the influence of slip condition, $\gamma = 0.5$

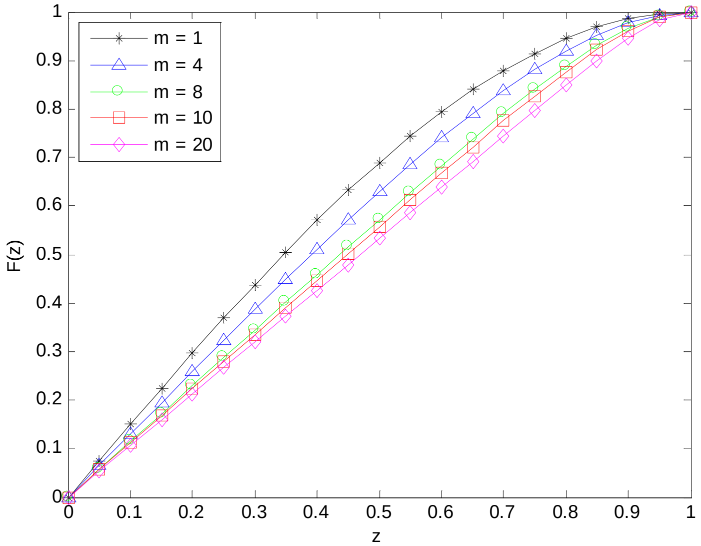

Figure 3. Effects of magnetic field parameter on the flow behavior of the fluid for no-slip condition

In order to get an insight into the problem, the effects of pertinent flow, magnetic field and slip parameters on the velocity profile of the fluid are investigated. Figure 2 shows the effects of the magnetic field parameter, Hartmann number m on the velocity of the fluid under the influence of slip condition, while Figure 3 depicts the influence of the magnetic field parameter on the velocity of the fluid under no-slip condition. It can be inferred from the figures that the velocity of the fluid increases with increase in the magnetic parameter under slip condition while an opposite trend was recorded during no-slip condition, as the velocity of the fluid decreases with increase in the magnetic field parameter under the no slip condition. The magnetic field plays the role of a resistance contributed to by the magnetic pressure field component of Lorentz force. The observed decrease in the velocity of the fluid as magnetic field increases under no-slip condition is due to the fact that the applied transverse magnetic field produces a damping or retarding force in the form of Lorentz force. As the value of magnetic parameter M increases, the retarding body force enhances and consequently the velocity reduces. The physical significance of this behavior is that the Lorentz force is a frictional resistive force which opposes the fluid motion and consequently, reduces the velocity of fluid flow. Under this scenario, the boundary layer thickness becomes thicker for a stronger magnetic field.

Figure 4 shows the influence of the slip parameter γ on the fluid velocity. By increasing γ, it is observed that the velocity of the fluid increases. The impact of slip conditions significantly enhances the velocity profile in the presence and absence of Hartmann number.

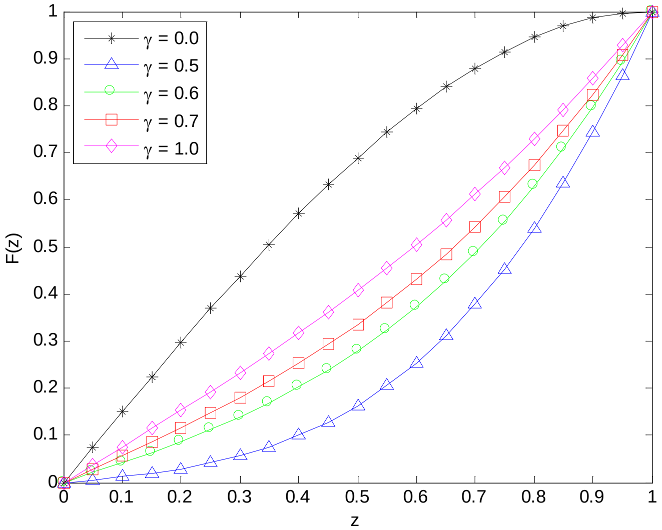

Figure 4. Effects of slip parameter on the flow behavior of the fluid

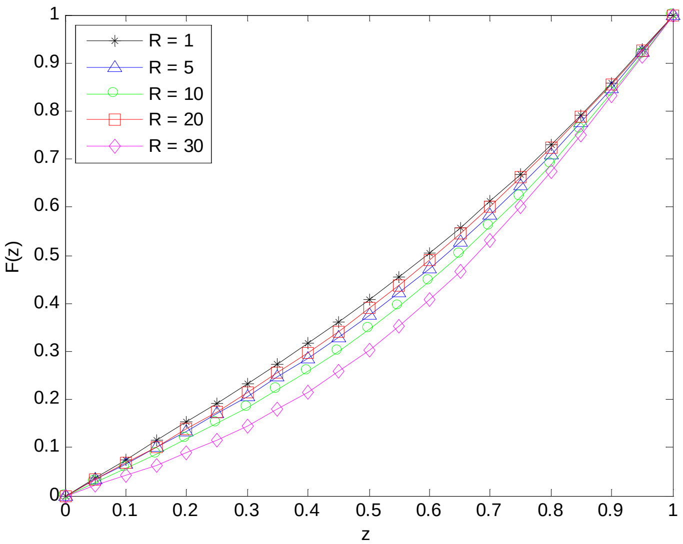

Figure 5. Effects of Reynolds number on the flow behavior of the fluid under the influence of slip condition

Figure 5 presents the effects of Reynolds number on the velocity of the fluid. It is observed from the figure that by increasing the value R, the velocity of the fluid decreases. Also, the flow increases significantly towards the center of the pipe, as depicted in the figure. The observed behavior of fluid velocity for different Reynolds number is because the flow toward the center becomes greater to make up for the space and consequently the fluid velocity also becomes greater near the center.

5. Conclusion

In this work, the homotopy perturbation method has been used to analyze steady two-dimensional axisymmetric flow of an incompressible viscous fluid under the influence of a uniform transverse magnetic field with slip boundary condition. Effects of pertinent flow, magnetic field and slip parameters have been investigated. It was established from the results that the velocity of the fluid increases with increase in the magnetic parameter under slip condition while the velocity of the fluid decreases with increase in the magnetic field parameter under the no slip condition. By increasing the slip parameter, the velocity of the fluid increases and the fluid velocity decreases as the Reynolds number increases. The approximate analytical solution has been verified by comparing the results of the approximate analytical methods with the numerical method using Runge-Kutta coupled with shooting method. It is hoped that this study will enhance and advance the understanding of axisymmetric squeezing flow of viscous fluid under no-slip and slip conditions.

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