

PORTABLE TENT STRUCTURE DESIGNS FOR FLOOD PROTECTION

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*Dedicated to Professor Barna Szabó on the occasion of his eightieth birthday
and to Professor Imre Kozák on the occasion of his eighty-fifth birthday*

Abstract. In this paper, we present two simple arrangements of portable tent structures for flood protection. In the first, an inflated cylinder lifts the top of the textile as the water level rises, while in the second the textile supporting the water pressure of the tide is hung on a steel framework and is filled with water previously. We give a numerical solution for the nonlinear system of equations of the compatibility and equilibrium conditions, and also present a complete analytic solution to the boundary value problem of the shape of an inextensible, weightless, prismatic textile, similar to the sessile drop problem. After discussing the mechanical behavior of the structures, different geometries capable of withstanding a certain water height are determined by a software developed for this specific purpose enabling the design for optimum. Furthermore, the basic rules of thumb are also formulated, aiding the preliminary conceptual design.

Mathematical Subject Classification: 65H05, 65B99

Keywords: Portable tent, flood protection, sessile drop, elastica, nonlinear pendulum, analytic solution

1. INTRODUCTION

Due to climate change, peak water levels more and more often exceed the height of the existing artificial embankment dams, causing ever increasing difficulties in flood-prone, inhabited areas near regulated rivers. Since the most common current emergency measures involve laying sandbags thus requiring considerable manpower, new methods are desired. In the first two chapters we present two possible alternatives, being fast and easy not only to transport but to construct as well, while in the last chapter we summarize the parametric representation of the directrix shape of a weightless, inextensible, prismatic membrane loaded by static water pressure, and we also derive the closed form equation of the area between the curve and an arbitrary vertical line. These formulations involve computationally burdensome elliptic

functions and their inverses; therefore, solving directly the governing boundary value problem by a numerical algorithm is also a good alternative.

2. FIRST ARRANGEMENT

The international Inflator project [1], managed by the MFKK Invention and Research Center and funded by the EU 7th Framework Programme, aimed to develop novel, affordable mobile flood defense systems. One of the many possibilities presented there involves a self-erecting textile. As participants, we developed a computer program capable of determining the static and geometric properties of the structure [2]. The detailed description of the algorithm and the analysis of the structural behavior are given in this section.

2.1. Geometry. This portable structure, mounted on the vertical crown of the existing embankment dam, solely consists of tensile elements. The first component is an inflated cylinder of radius r with diaphragms dividing its length into several cells and an additional active monitoring system for over-pressure control. The second is a sheet of impermeable textile skirt attached to a generatrix of the cylinder. The third component involves several tie-backs of length l welded to the same generatrix at equal distances along the length of the dam. The latter two are anchored to the foundation, a common fixed point on the horizontal crown of the embankment dam. A representative equilibrium cross-section is depicted in Fig. 1. We describe the setup in the x-y coordinate system. The main input parameters of the structure geometry are the BCDA textile section length (l_{BCDA}), the AB tie-back length (l_{AB}), the inflated cylinder radius (r), and the supported water level above crown level (h).

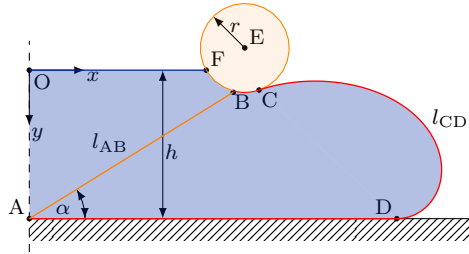


Figure 1. Cross-section of the 1st arrangement

2.2. Behavior of the structure. Initially the structure lies on the ground and is erected as the floating cylinder filled with air lifts the top of the textile with the rising water level, while the tie-back – capable of letting the water through – prevents the whole structure from collapsing. To avoid leakage, diaphragms and an electronic monitoring system are applied. The sufficiency of the anchorage at point A, although a critical issue of the general stability, is not addressed here.

2.3. Assumptions.

- The cross-section of the structure shows only negligible alterations in the longitudinal direction based on three-dimensional finite element simulation results, therefore it is sufficient to examine only the cross section in planar deformation state after linearly distributing the concentrated tie-back forces along directrix B.
- The over-pressure inside the inflated cylinder is high enough to maintain the circular cross-section, thus we handle it as a rigid body.
- The dimensions l_{BCDA} , l_{AB} , and r are constant, that is, all elements have tensile stiffness large enough for the loads to cause negligibly small strains.
- The weight of the textile, the tie-back, and the cylinder are negligible compared to the loads they bear.

2.4. Theoretical background. The internal forces and the equilibrium geometry of the arrangement are determined by a system of four nonlinear transcendental equations. The first two express the equilibrium of the cylinder, while the other two state the geometric compatibility of the textile.

2.4.1. Loads on the cylinder. Fig. 2 shows the loads acting on the inflated cylinder. These are:

1. T , the specific tensile force from the tie-back,
2. S , the specific force from the textile tangential to the cylinder, and
3. $p(\xi)$ the pressure from the water normal to the cylinder.

We describe the conformation of the whole arrangement by four parameters:

1. α , the inclination of the tie-back from the horizontal,
2. β , the angle between the horizontal and the BE line connecting the end of the tie-back and the center of the cylinder,
3. δ , the angle between the BE line and the CE line connecting the point where the textile separates from the cylinder and the center point, and
4. T , the specific force in the tie-back.

In terms of these parameters the coordinates of the center of the cylinder (E) are

$$x_E = l_{AB} \cos \alpha + r \cos \beta, \quad y_E = h - l_{AB} \sin \alpha - r \sin \beta, \quad (2.1)$$

the coordinates of the point where the canvas separates from the cylinder (C) are

$$x_C = x_E - r \cos(\beta + \delta), \quad y_C = y_E + r \sin(\beta + \delta), \quad (2.2)$$

the inclination of the tangent of the textile from the horizontal at point C is

$$\varphi_C = \frac{\pi}{2} - \beta - \delta, \quad (2.3)$$

the inclination of the FE line connecting the point where the water level intersects the cylinder and the center of the cylinder from the horizontal is

$$\gamma = -\arcsin \frac{y_E}{r}, \quad (2.4)$$

2.4.3. *Geometric compatibility of the textile.* The remaining two condition equations:

$$\begin{aligned} f_3(\alpha, \beta, \delta, T) &= l_{\text{BCDA}} - \delta \cdot r - l_{\text{CD}} - x_{\text{D}} = 0, \\ f_4(\alpha, \beta, \delta, T) &= h - y_{\text{D}} = 0, \end{aligned} \quad (2.9)$$

where l_{CD} , x_{D} , and y_{D} are calculated through (4.15), (4.20), and (4.9)₂, respectively, stem from the compatibility conditions of the textile, whose loaded shape determination is described in Section 4.

2.4.4. *The nonlinear system of equations.* To summarize (2.8) and (2.9), we write:

$$\mathbf{f}(\mathbf{x}) = 0, \quad \text{where} \quad \mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}, \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} \alpha \\ \beta \\ \delta \\ T \end{bmatrix}. \quad (2.10)$$

We solve (2.10) by a target code applying the Jacobi method implemented in MATLAB 2011a (The MathWorks Inc., Natick, Massachusetts, United States). As stated in (4.7), to find a physically admissible solution that prevents the textile from rising above water level, that is, to prevent inflection in the directrix, the domain of $\mathbf{f}$ is restricted so that the Eötvös number exceeds the critical value of 1.

2.5. Analysis of the structure.

2.5.1. *Water level elevation.* As an illustration, the equilibrium geometries of the arrangement $l_{\text{AB}} = 1.1$ m, $l_{\text{BCDA}} = 3.1$ m, $r = 0.2$ m are shown in the frames of Fig. 3 for water level rising by 10 centimeters. The corresponding specific tensile force increases in the tie-back (T) and in the textile (S), along with the tie-back inclination (α), are shown in Fig. 4.

2.5.2. *Maximal supported water level.* The maximal supported water level h is a function of l_{AB} , l_{BCDA} , and r together describing the geometry of a given arrangement. This extremal state is defined by $\gamma = -\pi/2$. We first consider the value of r as given and instead of δ we choose φ_{C} as the independent variable, constraining $y_{\text{C}} = r(1 + \cos \varphi_{\text{C}})$. To reach the desired $h = y_{\text{D}}$, which due to (4.7)₁ and (4.9)₂ is smaller than

$$h_{\text{max}} = \sqrt{\frac{2r^2(1 + \cos \varphi_{\text{C}})^2}{1 - \cos \varphi_{\text{C}}}}, \quad (2.11)$$

S is found from (4.9)₂ to be

$$S = \frac{\rho g (h^2 - y_{\text{C}}^2)}{2(1 + \cos \varphi_{\text{C}})}. \quad (2.12)$$

φ_{C} uniquely describes F_x and F_y , the components of the water pressure resultant as shown in Fig. 5 and in (2.13):

$$F_x = \frac{\rho g r^2}{2} (1 + \cos \varphi_{\text{C}})^2, \quad F_y = \frac{\rho g r^2}{2} [\varphi_{\text{C}} - \pi + \sin \varphi_{\text{C}} (2 + \cos \varphi_{\text{C}})]. \quad (2.13)$$

Through the equilibrium of the cylinder, the tie-back position is determined via

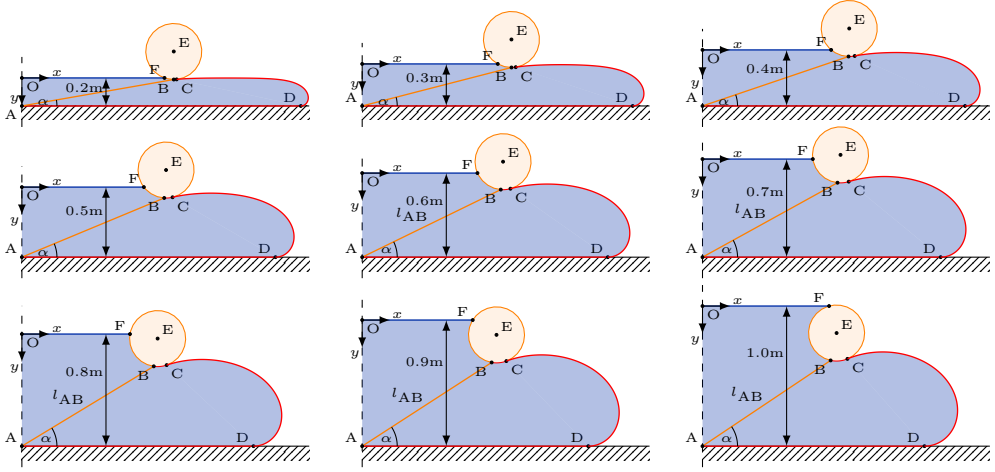


Figure 3. Frames of the equilibrium geomery of the arrangement $l_{AB} = 1.1$ m, $l_{BCDA} = 3.1$ m, $r = 0.2$ m in case of the water level rising by 10 cm in each step.

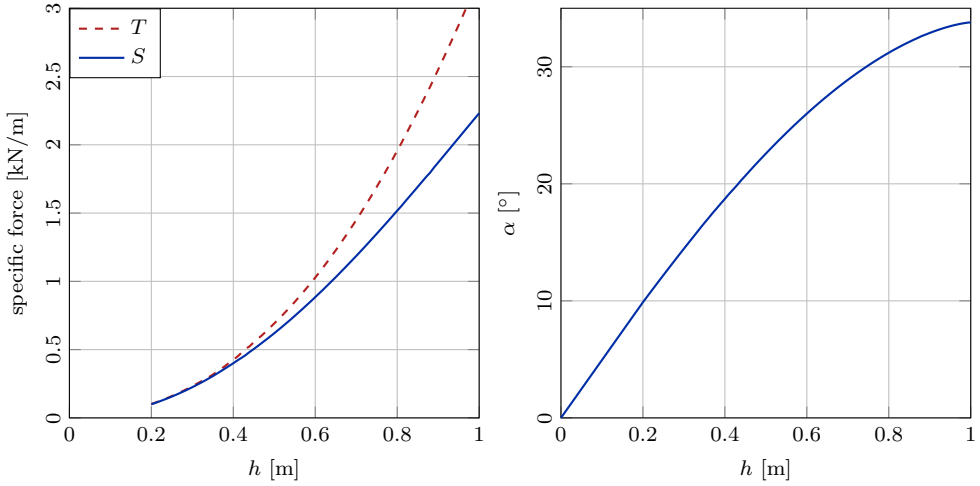


Figure 4. Specific tensile forces in the tie-back (T) and in the textile (S) (left), and the tie-back inclination (α) as a function of the rising water level (h)

$$\alpha = \arctan \frac{-S \sin \varphi_C - Fy}{S \cos \varphi_C + Fx}, \quad \beta = \alpha + \arcsin \frac{Fy \cos \alpha + Fx \sin \alpha}{Fy \cos \varphi_C - Fx \sin \varphi_C}, \quad (2.14)$$

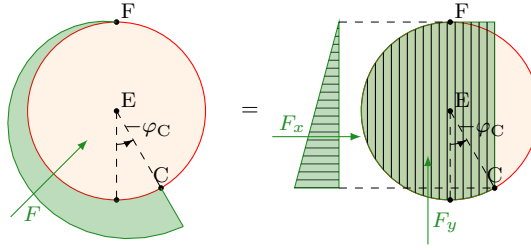


Figure 5. Decomposition of the water pressure resultant exerted on the cylinder in case of the highest possible supported water level

yielding the tie-back and textile lengths:

$$l_{AB} = \frac{h - r(1 + \sin \beta)}{\sin \alpha}, \quad l_{BCDA} = \delta \cdot r + l_{CD} + x_D, \quad (2.15)$$

where δ , l_{CD} and x_D are calculated via (2.3), (4.15) and (4.20), respectively.

2.6. Preliminary design.

We have now the possible geometric parameters characterizing the extremal design state of the structure. For a given r , their relation describes a surface in the three-dimensional (l_{AB} , l_{BCDA} , h) space, the contours of which are shown in Fig. 6 by blue lines, where it becomes possible to optimize the cost of the arrangement. Usually the textile length is to be minimized, both for financial reasons and because the top of the existing dam it is going to be placed on offers limited space, consequently the optimal designs are extremal points of the level lines forming the red curve also shown in Fig. 6. Agreeing upon this condition, we present the design method to define a geometry which is capable of withstanding a water height h :

1. We assume that h includes the uncertainty of the water level and the surface wave amplitudes as well, hence reducing the problem to a quasi-static investigation.
2. As Fig. 7 shows, the tie-back and textile lengths decrease radically with the increasing cylinder radius, the critical design parameter, chosen to be the highest feasible value.
3. l_{AB} and l_{BCDA} are concluded from Fig. 7.
4. S and T , the specific textile and tie-back forces are found from Fig. 8.
5. For designing the anchorage foundation, the tie-back inclination angle is shown in Fig. 9.

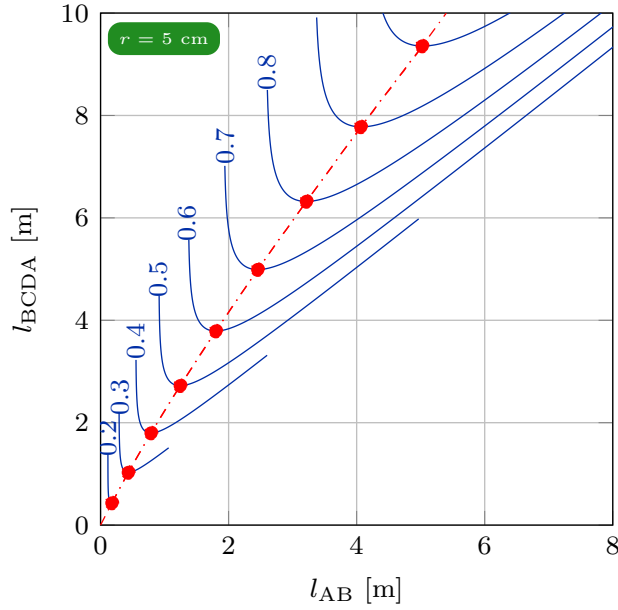


Figure 6. Blue level lines of the maximal supported water level, h [m] for a given cylinder radius ($r = 5$ cm); red curve is the optimal choice of the geometric parameters.

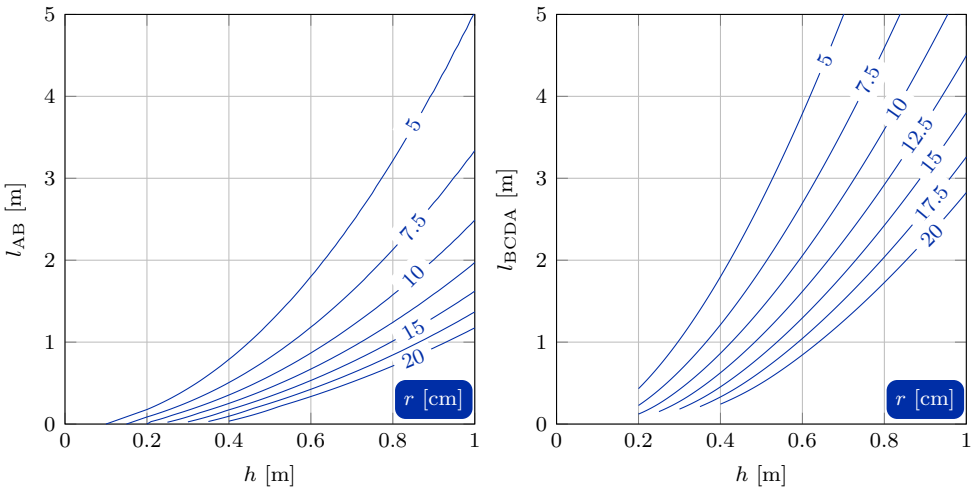


Figure 7. Tie-back and textile lengths as functions of the cylinder radius and maximal supported water level

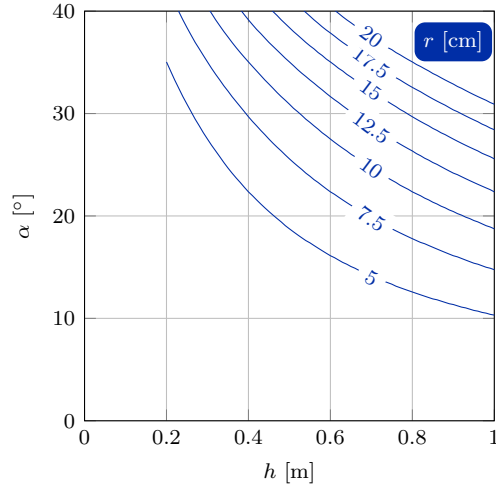


Figure 9. Extremal tie-back inclination angle as functions of the cylinder radius and maximal supported water level

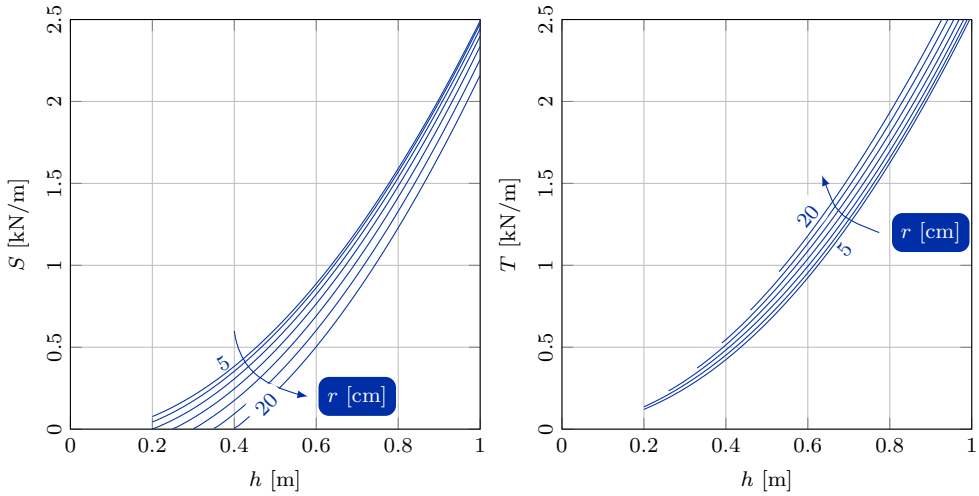


Figure 8. Specific textile and tie-back forces as functions of the cylinder radius and maximal supported water level

3. SECOND ARRANGEMENT

The second scenario was proposed to life by Airdome-Mobildam Kft. who approached us to analyze the structure presented in this chapter. Similarly to the first problem,

3.4. Unloaded, erected textile shape. Without the rising water load the erected structure is symmetric, therefore we only present the right half ($B_1C_1D_1$) of the arrangement $H = 1.6$ m, $d = 5$ cm and $L = 6$ m in Fig. 11 with internal water level rising by 20 centimeters. The whole textile is smooth, stretches straight in segment B_1C_1 , is supported by the ground in D_1O and takes the loaded shape defined in Section 4 in segment C_1D_1 . Note that the horizontal dimension of the structure

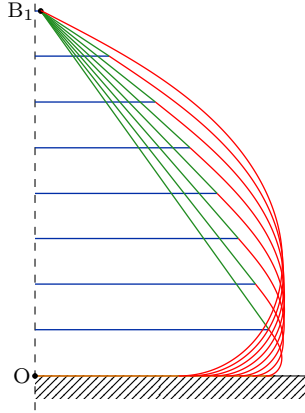


Figure 11. Unloaded, erected textile shape during the fillup process with internal water level rising by 20 centimeters

remains virtually unaltered during the process, thus the approximate formula for the preliminary design to calculate the horizontal space occupied is:

$$t_a = \frac{L^2 - 4H^2 - d^2}{2(L - d)}. \quad (3.1)$$

3.4.1. Governing equation. The textile shape results from Section 4, while the geometric compatibility of the structure as a whole for a given h_i internal water level is fulfilled by adjusting S_1 in

$$f_1(S_1) = 2 \left(\frac{H - h_i}{\sin \varphi_1} + l_{C_1D_1} + x_{D_1} \right) - L = 0, \quad (3.2)$$

where $l_{C_1D_1}$ and x_{D_1} are found from (4.15) and (4.20), respectively. Based on the restriction on the Hamiltonian of the system ($H_0 < 0$ see Section 4.2), or – in a mechanically more expressive form – the horizontal equilibrium of a textile section, the viable values of S_1 are taken from $(S_{1,\min}, \infty)$, where

$$S_{1,\min} = \frac{\rho g h_i^2}{4}. \quad (3.3)$$

We obtain the solution of (3.2) by the bisection method, consequently this unbounded interval is transformed to the bounded interval of $s_1 \in (0, 1]$ by introducing

$$s_1 = \frac{S_{1,\min}}{S_1}. \quad (3.4)$$

3.4.2. *Design parameters.* For calculating the necessary strength and stability of the frame the support force components are to be determined. In case of our example we illustrate them in Fig. 12 as a function of the internal water level elevation.

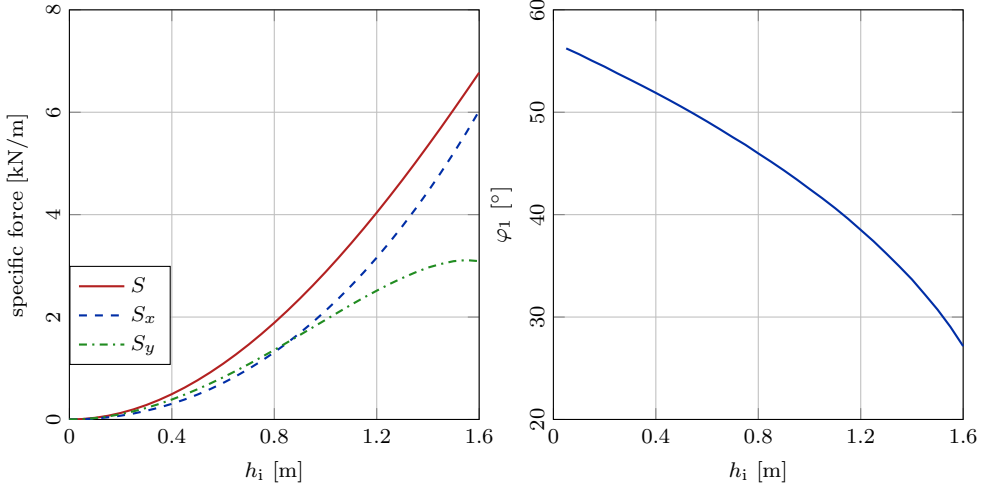


Figure 12. Specific tensile forces and its components in the textile (left), and initial inclination of the textile (right) as functions of the rising internal water level

3.5. **Loaded geometry.** The textile still remains smooth, stretches straight in segment B_2C_2 , is supported by the ground in ED_1 , takes the loaded shape defined in Section 4 in segment C_2D_2 , and forms a circle section in D_2E , the radius (R) and central angle (ϕ) of which is:

$$\begin{aligned} R &= \frac{S_2}{\varrho g (h_i - h_s)}, \\ \phi &= \arccos \frac{R - h_s}{R}. \end{aligned} \quad (3.5)$$

3.5.1. *Governing equation.* The geometric compatibility for a given h_i and h_s is again met by adjusting S_2 in the cross-sectional length error:

$$f_2(S_1) = 2 \left(\frac{H - h_i}{\sin \varphi_2} + l_{C_2D_2} + R\phi + x_{D_2} - R \sin \phi \right) - L = 0, \quad (3.6)$$

where $l_{C_2D_2}$ and x_{D_2} are found by evaluating at $\varphi = \pi - \phi$ the functions (4.13) and (4.19), respectively. Similarly to the unloaded case, the solution is obtained by the bisection method, thus the unbounded interval of $S_2 \in (S_{2,\min}, \infty)$, where

$$S_{2,\min} = \frac{\varrho g (h_i^2 - h_s^2)}{4} \quad (3.7)$$

is transformed to the bounded interval of $s_2 \in (0, 1]$ by introducing

$$s_2 = \frac{S_{2,\min}}{S_2}. \quad (3.8)$$

3.6. Rise of the supported water level. Once the desired internal water level is reached, its volume (found from (4.21) and shown in Fig. 13) does not change throughout the increasing loading of the rising water level. Numerically, after the two

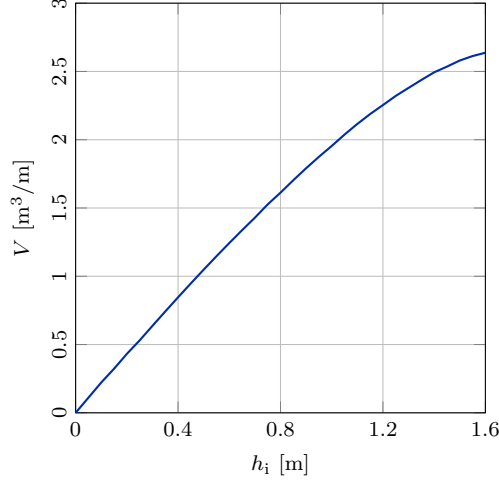


Figure 13. Specific volume of the infill against its rising level

internal iterations of S_1 and S_2 for the geometric compatibility of the two sides, an outer bisection iteration adjusts h_i to match the volume requirement:

$$f_3(h_i) = 2V(\varphi_C, \pi, S) - V(\varphi_{C_1}, \pi, S_1) + V(\varphi_{C_2}, \varphi_{D_2}, S_2) + \frac{R^2\phi}{2} - \frac{R^2\sin 2\phi}{4} + R(1 - \cos \phi)(x_{D_2} - R\sin \phi) = 0, \quad (3.9)$$

where V is defined in (4.21), arguments with indices 1 and 2 represent the free and the loaded side, respectively while the symmetric unloaded geometry is referred to by subscripts without any index. Fixing the infill volume at the value when $h_i = 1.3$ m in the unloaded state, the increasing h_s deforms the cross-section shown in the frames of Fig. 14. Figure 15 depicts the tension (S_2) in the loaded textile decreasing with h_s .

3.7. Conclusions.

- The structure acts as a gravity retaining wall.
- The symmetry of the erected structure is violated by the supported rising water pressure.

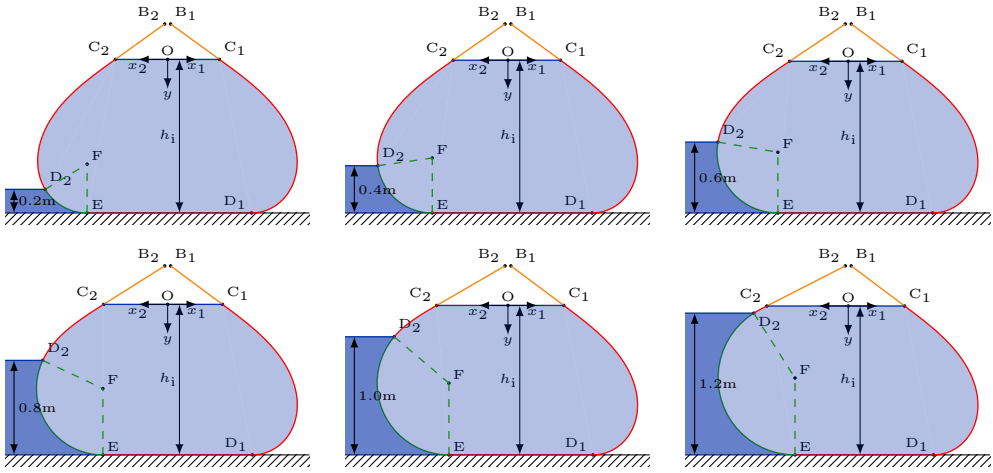


Figure 14. Frames of the structure deformation as the supported water level rises by 20 centimeters

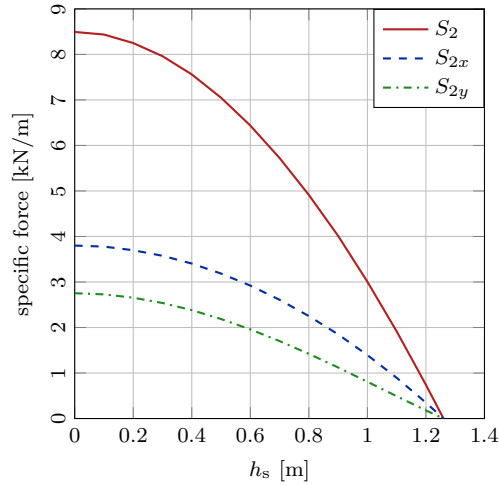


Figure 15. Specific tensile force and its components in the textile loaded by the rising supported water level

- Counter-intuitively, instead of pushing the structure away from the riverbed, the external water pulls it closer due to the uplift.
- The internal water level decreases, although insignificantly (by 5 cm in the presented example, see Figure 15, where the specific tensile force becomes zero at $h_s = 1.25$ cm).

- The textile tension in the supporting side decreases to zero as h_s reaches h_i , when the supporting textile loosens and its shape becomes undefined.
- The properties of the side away from the river depend only on the internal water level, which is virtually unaltered, thus its shape and tension agree with the unloaded state.
- $h_s = h_i$ is the critical state from the point of view of stability, when total load of the rising water acts only on the outer textile part, while the balancing tension in the other side of the textile is missing.

3.8. Preliminary design. The preliminary design for an expected $h_{s,\max}$ starts with prescribing H and d for the framework and t , the necessary minimal length where the textile touches the ground at the time of the maximal tidal level to prevent the river flooding out and to provide the overall stability of the structure; and results in the critical design force descriptors from the point of view of the framework: S_1 , the specific force in the outer textile part and φ_{C_1} , the inclination of the textile tangent at point C_1 . In Fig. 16, we present the design charts supporting this simplified work flow. We consider the limit case when $h_i = 1.25h_{s,\max}$, and assume that $H = h_i$. Having agreed upon the safety ratio of 1.25 and infill ratio of 1, the designer chooses a suitable t , adjusts it by d and from the left graph on Figure 16 the necessary cross-sectional textile length is obtained alongside with S_1 as is φ_{C_1} from the chart on the right. For a given H , the increase in L causes a decrease in both S_1 and φ_{C_1} , thus not only because of the limited space available and cost efficiency reasons, but also because of overall structural stability issues, the shortest L is to be chosen meeting the criterion on t .

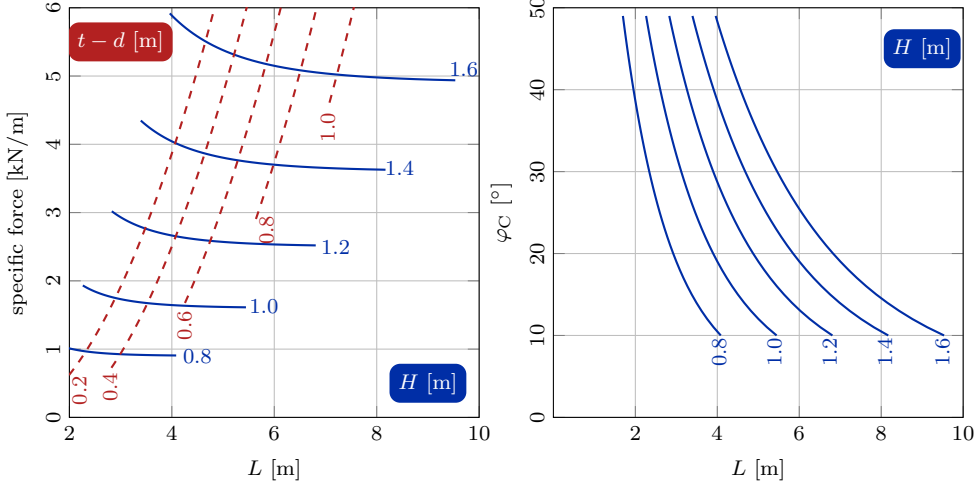


Figure 16. Dependence of the specific force in the outer textile part (S_1) (left) and the inclination of the textile tangent from the vertical at point C_1 (φ_{C_1}) (right) on the cross-sectional textile length (L) and the framework height (H) when the supported water height (h_s) is $0.8H$

4. SHAPE OF THE LOADED TEXTILE SECTION

Consider a prismatic, weightless, inextensible textile supporting static water pressure, the cross-section of which is shown in Fig. 17, where the water level is at $y = 0$. In this section we give the parametric functional description of this directrix (in the following referred to as fiber) shape and the closed form equation of the V area between the curve and the y axis. For similar structures, such as closed pressurized geomembrane tubes see [3] for the elastic case, [4] for stacked arrangement, [5] for infill with liquids of different density, [6] for compressible subgrade soil; or [7] for liquid-filled nonlinearly elastic shells.

4.1. Initial value problem. The initial value problem of ordinary differential equations describing the fiber shape is given by

$$\begin{aligned} x' &= \cos \varphi, & x(0) &= x_C, \\ y' &= \sin \varphi, & y(0) &= y_C, \\ \varphi' &= \frac{\varrho g}{S} y, & \varphi(0) &= \varphi_C, \end{aligned} \quad (4.1)$$

where x and y are the horizontal and vertical coordinates respectively, φ is the inclination of the tangent from the positive x axis in clockwise direction, ϱ is the water density, g is the gravitational acceleration, S is the specific tensile force in the fiber, and prime denotes the derivative with respect to the arch length parameter (λ). The

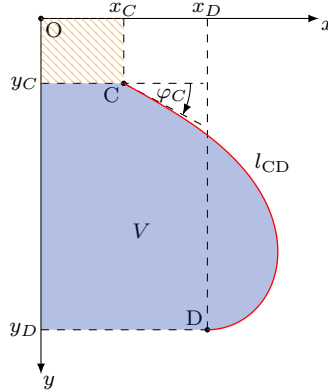


Figure 17. Boundary value problem of the textile shape loaded by static water pressure

first two equations are the compatibility conditions and the third is the tangential equilibrium statement. Since the fiber weight is negligible and the water pressure is normal to the curve, the equilibrium condition in the normal direction prescribes the specific tension (S) in the fiber being constant. By eliminating y from the last two ODEs of (4.1), we simplify the problem to

$$\varphi'' = \omega^2 \sin \varphi, \quad \varphi'(0) = \omega^2 y_C, \quad \varphi(0) = y_C, \quad \omega^2 = \frac{\rho g}{S}, \quad (4.2)$$

which is analogous to the nonlinear pendulum equation [8] except for the sign change. The same equation governs the elastica [9] and capillarity problems and if we consider S to be the surface tension, we arrive at the problem of sessile drops.

4.2. The first integral. We can define the corresponding Lagrangian:

$$L = \varphi'^2 + 4\omega^2 \sin^2 \frac{\varphi}{2}, \quad (4.3)$$

which does not depend explicitly on the arch-length, thus following Noether's theorem [10] we construct a conserved quantity:

$$H = \varphi'^2 - 4\omega^2 \sin^2 \frac{\varphi}{2} = \text{const.} \quad (4.4)$$

This – speaking in mechanical terms – states the horizontal equilibrium of a textile section. At a certain depth the first term is proportional to the horizontal hydrostatic load resultant and the second is proportional to the horizontal tensile force in the textile, resulting in the alternate form of (4.4):

$$\tilde{H} = \frac{\rho g y^2}{2} + S \cos \varphi = \text{const.} \quad (4.5)$$

Henceforth we work with the form of (4.4). This translational symmetry, analogous to the energy or the Hamiltonian of the pendulum moving in a conservative field, suggests scrutinizing the periodic phase space and its two qualitatively different trajectories

DTCD ($0 < H = \text{const.}$) and SUVS ($0 > H = \text{const.}$) separated by the dash-dotted separatrix ($H = 0$) in Fig. 18. We introduce two constants:

$$H_0 = \omega^4 y_C^2 - H_p \sin^2 \frac{\varphi_C}{2}, \quad H_p = 4\omega^2, \quad (4.6)$$

which correspond to the total energy and the maximal potential energy of the pendulum, respectively.

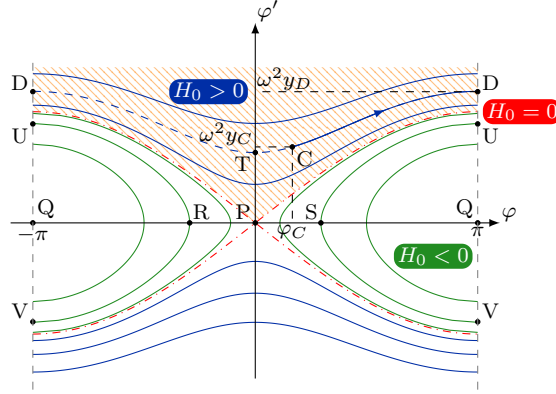


Figure 18. Phase space of the Hamiltonian

In the first arrangement (Section 2), we allow only the curves above the upper separatrix since this restricts the y coordinate to be positive, that is the fiber does not rise above the water level, and has no inflection, which is the only physically acceptable scenario, analogous to the overturning pendulum. The necessary criterion is $H_0 > 0$, which yields an inequality constraint to the boundary conditions given by

$$y_C > y_l, \quad \text{where} \quad y_l = \sqrt{\frac{2(1 - \cos \varphi_c) S}{\varrho g}}, \quad \text{or} \quad (4.7)$$

$$Eo > 1, \quad \text{where} \quad Eo = \frac{\varrho g y_c^2}{2(1 - \cos \varphi_c) S}.$$

Here y_l is the capillarity length and Eo is the Eötvös number, both measuring the relationship between the gravity and surface tension effects, and extensively used for analyzing capillarity phenomena such as the shape of pendant or sessile drops [11]. Now we are able to determine

$$y(\varphi) = \sqrt{y_C^2 + \frac{4}{\omega^2} \left(\sin^2 \frac{\varphi}{2} - \sin^2 \frac{\varphi_C}{2} \right)}, \quad (4.8)$$

which gives us the maximal and minimal textile heights denoted by points D and T in the phase space:

$$\begin{aligned} y_T &= \sqrt{y_C^2 - \frac{2}{\omega^2} (1 - \cos \varphi_C)}, \\ y_D &= \sqrt{y_C^2 + \frac{2}{\omega^2} (1 + \cos \varphi_C)}. \end{aligned} \quad (4.9)$$

In the second arrangement (Section 3), analogous to the oscillating pendulum, the only viable curves are between the separatrices ($H_0 < 0$) with initial conditions $\varphi' = 0$ and $\varphi > 0$, turning around the relations in (4.7), but leaving (4.8) unaltered.

4.3. Elliptic functions. In the following we use the definition of Legendre's elliptic integrals (4.10) and the Jacobi functions (4.11). These are the incomplete elliptic integrals of the first (F) and second (E) kind; the Jacobi amplitude (am), the sinus amplitudinis (sn), and cosinus amplitudinis (cn) functions:

$$F(\varphi|A) = \int_0^\varphi \frac{d\phi}{\sqrt{1 - A \sin^2 \phi}}, \quad (4.10)$$

$$E(\varphi|A) = \int_0^\varphi \sqrt{1 - A \sin^2 \phi} d\phi,$$

$$\begin{aligned} \text{am}(\lambda|A) &= F^{-1}(\lambda|A), \\ \text{sn}(\lambda|A) &= \sin[\text{am}(\lambda|A)], \\ \text{cn}(\lambda|A) &= \cos[\text{am}(\lambda|A)], \end{aligned} \quad (4.11)$$

4.4. Parametric description of the fiber. In the first scenario $H_p = 4\omega^2 > 0$, $H_0 > 0$, and $\varphi' > 0$; in the second $H_0 < -H_p \sin^2(\varphi_C/2) < 0$ and $\varphi' > 0$ since $\varphi \in [\varphi_C, \pi]$. Consequently, for both arrangements (4.4) results in the ODE:

$$\varphi' = \sqrt{H_0 + H_p \sin^2 \frac{\varphi}{2}}, \quad (4.12)$$

and its solutions:

$$\lambda(\varphi) = \text{sign}(H_0) \frac{2}{\sqrt{H_0}} F\left(\frac{\varphi}{2}|A\right) - \lambda_C, \quad \text{and} \quad \varphi(\bar{\lambda}) = 2\text{am}(\bar{\lambda}|A), \quad \text{where} \quad (4.13)$$

$$\bar{\lambda} = \text{sign}(H_0) \frac{\sqrt{H_0}}{2} (\lambda + \lambda_C), \quad \lambda_C = \text{sign}(H_0) \frac{2}{\sqrt{H_0}} F\left(\frac{\varphi_C}{2}|A\right), \quad A = -\frac{H_p}{H_0}. \quad (4.14)$$

Hence the length of the CD curve is

$$l_{CD} = \text{sign}(H_0) \frac{2}{\sqrt{H_0}} \left[F\left(\frac{\varphi_D}{2}|A\right) - F\left(\frac{\varphi_C}{2}|A\right) \right]. \quad (4.15)$$

Substituting (4.13)₂ into (4.8) gives

$$y(\bar{\lambda}) = \sqrt{y_C^2 + \frac{4}{\omega^2} \left[\text{sn}^2(\bar{\lambda}|A) - \sin^2 \frac{\varphi_C}{2} \right]}. \quad (4.16)$$

Merging (4.2)₁ and (4.13)₂, we get

$$x' = \cos [2\text{am}(\bar{\lambda}|A)] = 1 - 2\text{sn}^2(\bar{\lambda}|A), \quad (4.17)$$

and its solution

$$\begin{aligned} x(\bar{\lambda}) = x_C + \frac{4H_0 + 2H_p}{\sqrt{H_0}H_p} (\bar{\lambda} - \bar{\lambda}_C) + \\ + \frac{4\sqrt{H_0}}{H_p} \{E[\text{am}(\bar{\lambda}_C|A)|A] - E[\text{am}(\bar{\lambda}|A)|A]\}, \end{aligned} \quad (4.18)$$

where $\bar{\lambda}_C = \sqrt{H_0}\lambda_C/2$, or in terms of φ

$$\begin{aligned} x(\varphi) = x_C + \text{sign}(H_0) \frac{4H_0 + 2H_p}{\sqrt{H_0}H_p} \left[F\left(\frac{\varphi}{2}|A\right) - F\left(\frac{\varphi_C}{2}|A\right) \right] + \\ + \text{sign}(H_0) \frac{4\sqrt{H_0}}{H_p} \left[E\left(\frac{\varphi_C}{2}|A\right) - E\left(\frac{\varphi}{2}|A\right) \right]. \end{aligned} \quad (4.19)$$

We also give the x coordinate of point D:

$$x_D = x_C + l_{CD} \frac{H_p + 2H_0}{H_p} + \text{sign}(H_0) \frac{4\sqrt{H_0}}{H_p} \left[E\left(\frac{\varphi_C}{2}|A\right) - E\left(\frac{\varphi_D}{2}|A\right) \right]. \quad (4.20)$$

4.5. Enclosed area. The area between the y axis and the curve, when $\varphi \in [\varphi_C, \varphi_D]$ (shaded area in Fig. 17), can be deduced using Green's theorem, partial integration, variable change, (4.16) and (4.1)_{1,3} in this order, as shown in (4.21).

$$\begin{aligned} V(\varphi_C, \varphi_D, S) &= \int_{y_C}^{y_D} x \, dy = \int_0^{l_{CD}} xy' \, d\lambda = [xy]_C^D - \int_0^{l_{CD}} x'y \, d\lambda = \\ &= [xy]_C^D - \int_{\varphi_C}^{\varphi_D} \frac{x'y}{\varphi'} \, d\varphi = x_D y_D - x_C y_C - \int_{\varphi_C}^{\varphi_D} \frac{\cos \varphi y}{\omega^2 y} \, d\varphi = \\ &= x_D y_D - x_C y_C - \frac{1}{\omega^2} \int_{\varphi_C}^{\varphi_D} \cos \varphi \, d\varphi = \\ &= x_D y_D - x_C y_C + (\sin \varphi_C - \sin \varphi_D) \frac{S}{\rho g}. \end{aligned} \quad (4.21)$$

4.6. Conclusion. In this section we derived the parametric functional description of the cross-sectional shape and the explicit function of the enclosed area of an inextensible weightless prismatic textile subjected to water pressure loading and constant specific tensile force with known initial point coordinates (x_C, y_C) and prescribed initial tangent line inclination (φ_C) . As a summary, we give here the most important

equations of the fiber position:

$$\begin{aligned}
 x(\varphi) = & x_C + \operatorname{sign}(H_0) \frac{4H_0 + 2H_p}{\sqrt{H_0}H_p} \left[F\left(\frac{\varphi}{2} | A\right) - F\left(\frac{\varphi_C}{2} | A\right) \right] + \\
 & + \operatorname{sign}(H_0) \frac{4\sqrt{H_0}}{H_p} \left[E\left(\frac{\varphi_C}{2} | A\right) - E\left(\frac{\varphi}{2} | A\right) \right], \\
 y(\varphi) = & \sqrt{y_C^2 + \frac{4S}{\rho g} \left(\sin^2 \frac{\varphi}{2} - \sin^2 \frac{\varphi_C}{2} \right)},
 \end{aligned} \tag{4.22}$$

and the area enclosed by the y axis and the curve:

$$V(\varphi) = xy - x_C y_C + (\sin \varphi_C - \sin \varphi) \frac{S}{\rho g}, \tag{4.23}$$

the parameter (φ) being the inclination angle.

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